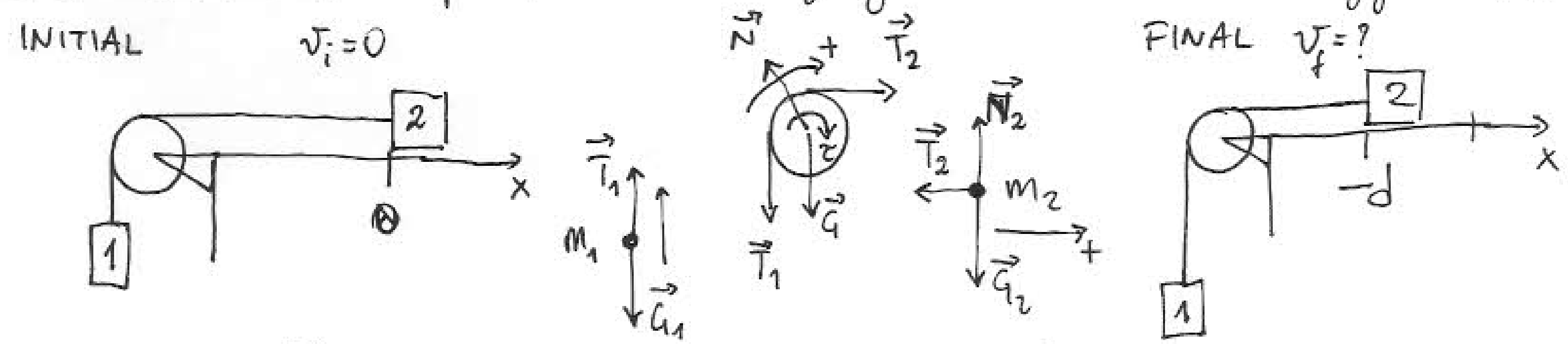
MID TERM EXAM N. 3

QUESTION 1

Question (a) asks for a speed, hence we are going to use the kinetic energy theorem.



$\vec{G}$, $\vec{N}$, $\vec{G}_2$ and $\vec{N}_2$ do not do any work. The tensions $\vec{T}_1$ and $\vec{T}_2$ do work of opposite sign in the different masses so their contributions is null. Hence:

$$W = W_{G_1} + W_\tau = m_1 g \cdot d - \frac{\tau_f}{R} \cdot d$$

$\frac{\tau_f}{R}$ being the friction force because τ_f is the friction torque.

The kinetic energy theorem reads: $\Delta K = \frac{1}{2}(m_1+m_2)v_f^2 + \frac{1}{2} I \left(\frac{v_f}{R}\right)^2 = \left(m_1 g - \frac{\tau_f}{R}\right) \cdot d$, hence

$$v_f = \sqrt{\frac{2\left(m_1 g - \frac{\tau_f}{R}\right)}{m_1 + m_2 + \frac{I}{R^2}}}$$

Question (b) asks for a time, hence we have to apply the Newtonian method. From the force diagrams above, considering the equation along the direction of motion:

1) $a = \dfrac{T_1 - m_1 g}{m_1}$

2) $\alpha = \dfrac{\tau + (T_2 - T_1) \cdot R}{I}$

3) $a = \dfrac{-T_2}{m_2}$

On this equation, torques and moment of inertia are calculated with respect the axis or rotation. $\vec{G}$ and $\vec{N}$ do not contribute.

Knowing that $\alpha = \frac{a}{R}$, one can solve for a; at the end:

$$a = \frac{-m_1 g + \frac{\tau_f}{R}}{m_1 + m_2 + \frac{I}{R^2}}$$

Since the acceleration is constant the resulting motion is:

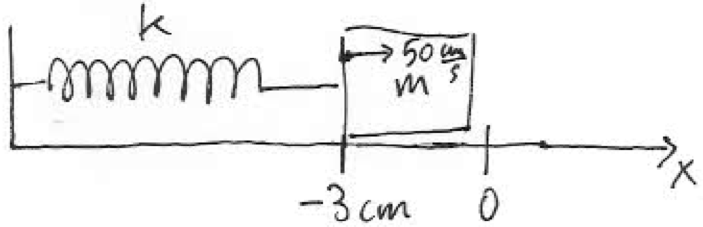
$$x(t) = x_0 + v_0 \cdot t + \frac{1}{2} a t^2$$

Assuming the initial event as the OTE, the motion becomes:

$x(t) = \frac{1}{2} a t^2$ and the time t_d at which $x(t_d) = -d$ is:

$$t_d = \sqrt{\frac{-2d}{a}} = \sqrt{\frac{2d\left(m_1 + m_2 + \frac{I}{R^2}\right)}{m_1 g - \frac{\tau_f}{R}}}$$

QUESTION 2



A mass attached to a spring, with no friction, is a simple harmonic oscillator. Its motion is in general of this type:

$$x(t) = A \cdot \cos(\omega t + \varphi)$$

where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{1}} = 5 \frac{rad}{s}$

A and φ are found from the initial conditions: $x(0) = -3cm$ $v(0) = 50 \frac{cm}{s}$
Deriving $x(t)$ with respect time one finds $v(t) = -A\omega \cdot \sin(\omega t + \varphi)$, hence:

$$\left.\begin{array}{l} x(0) = -0.03 = A \cdot \cos(\varphi) \\ v(0) = 0.50 = -A \cdot \omega \cdot \sin(\varphi) \end{array}\right\} \Rightarrow tg(\varphi) = \frac{0.50}{0.03\,\omega} \rightarrow \varphi = \tan^{-1}\left(\frac{50}{0.03 \cdot 5}\right) \cong 1.28 \text{ rad}$$

$$\text{and } A = -\frac{0.03}{\cos(1.28)} \cong -10.5 \text{ cm}$$

Answer to question (a):

$$x(t) = -10.5 \cdot \cos(5 \cdot t + 1.28) \text{ cm}$$
$$v(t) = 52.5 \cdot \sin(5t + 1.28) \frac{cm}{s}$$
$$a(t) = 2.63 \cdot \cos(5t + 1.28) \frac{m}{s^2}$$

answer to question (b)

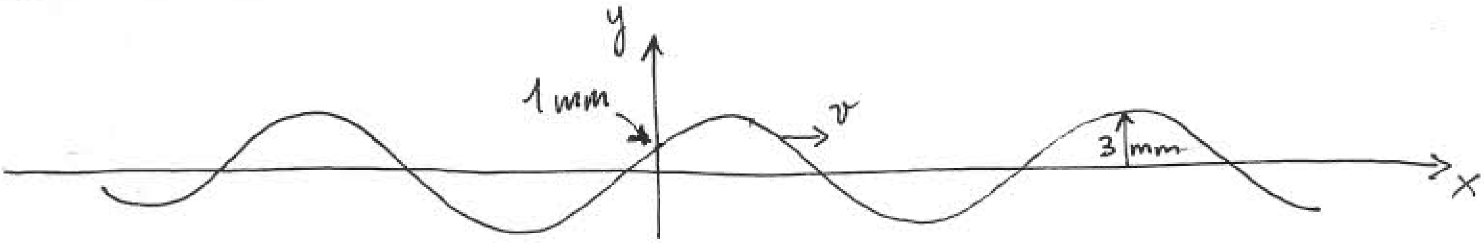
$$x_{max} = 10.5 \text{ cm}$$
$$v_{max} = 0.525 \frac{m}{s}$$
$$a_{max} = 2.63 \frac{m}{s^2}$$

The energy of this motion can be calculated as maximum potential or kinetic energy:

$$U_{max} = \frac{1}{2} k\, x_{max}^2 = \frac{1}{2} 25 \cdot (0.105)^2 = 0.138 \text{ J}$$

$$K_{max} = \frac{1}{2} m (A \cdot \omega)^2 = \frac{1}{2} \cdot 1 (0.105 \cdot 5)^2 = 0.138 \text{ J}$$

QUESTION N.3



We suppose the wave travelling to the right.

The general expression for an harmonic wave is :

$$y(x,t) = A \cdot \sin\left(kx - \omega t + \varphi\right)$$

Let's find the parameters:

$A = 3\text{ mm} = 3 \cdot 10^{-3}\text{ m}$

$\omega = 2\pi \cdot f = 2\pi \cdot 75 = 150\pi \ \frac{\text{rad}}{s}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{v \cdot T} = \frac{\omega}{v}$ but $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{50}{0.03}} \cong 40.8 \ \frac{m}{s}$ hence

$k = \frac{150\pi}{40.8} = 3.68 \cdot \pi \ m^{-1}$

Since $y(0,0) = 1\text{ mm} \rightarrow 1 = 3 \cdot \sin\varphi \rightarrow \varphi = \sin^{-1}\left(\frac{1}{3}\right) \cong 0.34$ rad

the expression of this wave is then $y(x,t) = 3 \cdot \sin\left(3.68\pi \cdot x - 150\pi \cdot t + 0.34\right)$ mm

The energy in 12λ is calculated as the maximum kinetic energy of a mass equal to $\mu \cdot 12 \cdot \lambda$, hence

$$E = \frac{1}{2}\mu \cdot 12 \cdot \lambda \cdot (A \cdot \omega)^2 = \frac{1}{2} 0.03 \cdot 12 \cdot \frac{2\cancel{\pi}}{3.68 \cdot \cancel{\pi}} \cdot \left(3 \cdot 10^{-3} \cdot 150 \cdot \pi\right)^2 = 0.719 \text{ J}$$

the power is

$$P = \frac{1}{2}\mu\,(A \cdot \omega)^2 \cdot v = \frac{1}{2} 0.03 \left(3 \cdot 10^{-3} \cdot 150\pi\right)^2 \cdot 40.8 \cong 1.22 \text{ W}$$