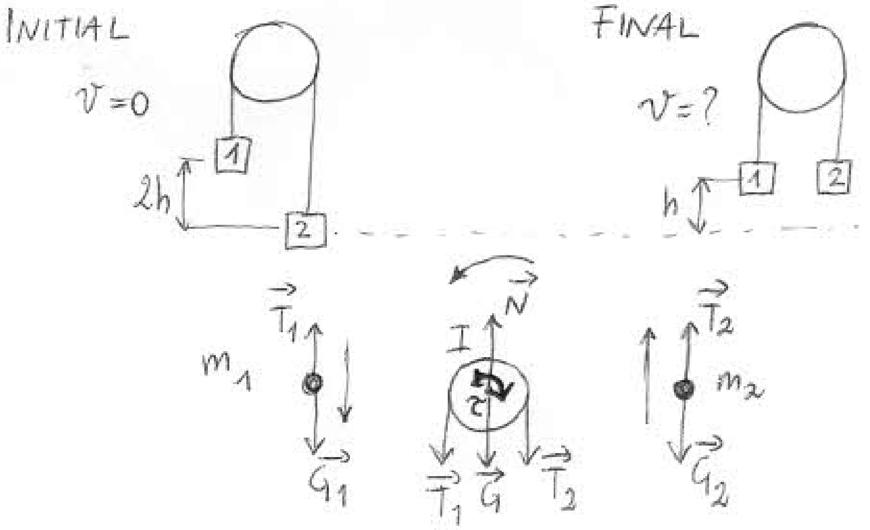
CLASS TEST 3

QUESTION 1



$\Delta K = W$

$W = W_{G_1} + W_\tau + W_{G_2} =$

$= m_1 g h - \tau/R \cdot h - m_2 g h$

hence :

$\Delta K = K_f - K_i = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 + \frac{1}{2} I \left(\frac{v_f}{R}\right)^2 =$

$= \left[(m_1 - m_2) g - \tau/R\right] h$

The works of T_1 and T_2 sum to zero because T_1 and T_2 are internal forces $\vec{N}$ and $\vec{G}$ do not produce work. The resulting speed is :

$$v_f = \sqrt{\frac{2\left[(m_1 - m_2) g - \tau/R\right] \cdot h}{m_1 + m_2 + \frac{I}{R^2}}}$$

To answer question (b) one needs to apply the Newton's method. Following the reference systems indicated in the force diagrams above, the 3 equations are :

1) $a = \dfrac{m_1 g - T_1}{m_1}$

2) $\alpha = \dfrac{-\tau + (T_1 - T_2) \cdot R}{I}$

3) $a = \dfrac{T_2 - m_2 g}{m_2}$

knowing that $\alpha = \frac{a}{R}$ one can solve for a :

$$a = \frac{(m_1 - m_2) g - \tau/R}{m_1 + m_2 + \frac{I}{R^2}} \rightarrow$$

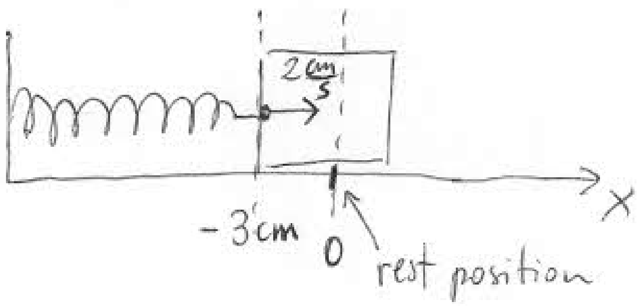
Since a = constant the motion is :

$$x = x_0 + v_0 \cdot t + \frac{1}{2} a t^2$$

Considering the mass 1, setting the OTE at its initial position, the motion is $X_1 = \frac{1}{2} a t^2$. The time t_h ~~to tra~~ required to travel a distance of h is then : $t = \sqrt{\frac{2h}{a}} \rightarrow$

$$\rightarrow t = \sqrt{\frac{2h\left(m_1 + m_2 + \frac{I}{R^2}\right)}{(m_1 - m_2) g - \tau/R}}$$

QUESTION 2



The general expression of the motion is :

$$x(t) = A \cdot \cos(\omega t + \varphi)$$

where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{1}} = 5 \ \frac{rad}{s}$

A and φ are found from the conditions on position and speed, which is :

$$v(t) = -A\omega \sin(\omega t + \varphi)$$

Assuming that the OTE is the one indicated in the picture :

$$\begin{cases} -0.03 = A \cdot \cos(\varphi) \\ 0.02 = -A \cdot \omega \cdot \sin(\varphi) \end{cases} \rightarrow \tan(\varphi) = \frac{0.02}{0.03 \cdot \omega} \Rightarrow \varphi = \tan^{-1}\left(\frac{2}{15}\right) = 0.133 \text{ rad}$$

$$\text{and } A = -\frac{0.03}{\cos(0.133)} = -3.03 \text{ cm}$$

the displacement is then $x(t) = -3.03 \cdot \cos(5 \cdot t + 0.133)$ cm

Speed and acceleration are $v(t) = 15.15 \sin(5 \cdot t + 0.133) \ \frac{cm}{s}$

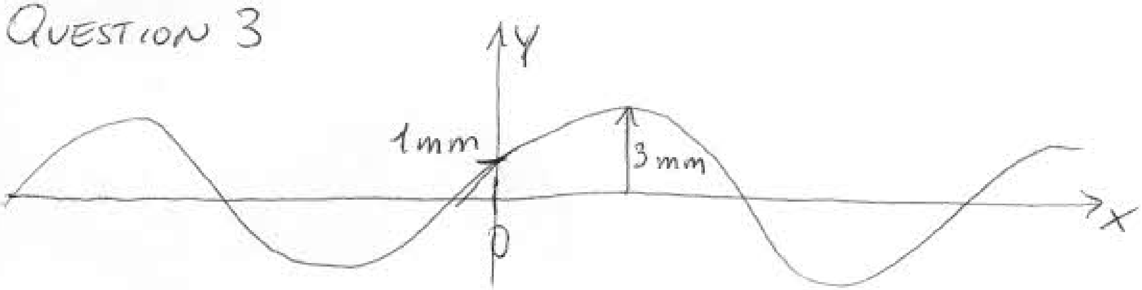
$$a(t) = 75.75 \cos(5 \cdot t + 0.133) \ \frac{cm}{s^2}$$

Maximum speed is $15.15 \ \frac{cm}{s}$ and maximum acceleration $75.75 \ \frac{cm}{s^2}$

the energy of the motion is $E = K_{max} = U_{max}$.

$$K_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} \cdot 1 \cdot 0.1515^2 = 0.0115 \text{ J}.$$

QUESTION 3



The general expression is $Y(x,t) = A\cdot \sin(k\cdot x - \omega t + \varphi)$

Some of the parameters are almost ready to be known, like ω:

$\omega = 2\pi f = 2\pi \cdot 75 = 150\pi \ \frac{rad}{s}$

$A = 3 \ mm = 3\cdot 10^{-3} \ m$

k is known through the speed. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{50}{0.03}} \cong 40.8 \ \frac{m}{s}$ and since

$v = \frac{\lambda}{T} = \frac{\omega}{k} \rightarrow k = \frac{\omega}{v} = \frac{150\pi}{40.8} = 3.68\pi \ m^{-1}$. For the last parameter, φ:

$Y(0,0) = 1 = 3\sin(\varphi) \Rightarrow \varphi = \sin^{-1}\left(\frac{1}{3}\right) \cong 0.34$ rad. The wave is:

$$Y(x,t) = 3\cdot \sin(3.68\pi\cdot x - 150\pi\cdot t + 0.34) \ mm$$

The energy in 1λ is $E_\lambda = \frac{1}{2}\mu\cdot\lambda\,(A\cdot\omega)^2 = \frac{1}{2}\mu\frac{2\pi}{k}(A\cdot\omega)^2 =$

$$= 0.03\cdot\frac{\cancel{\pi}}{3.68\cdot\cancel{\pi}}\cdot\left(3\cdot 10^{-3}\cdot 150\cdot\pi\right)^2 = 16.3 \ mJ$$

In 12λ the result for the energy is $12\cdot 16.3 mJ = 195.6 \ mJ$

The power is: $P = \frac{1}{2}\mu(A\omega)^2\cdot v = \frac{1}{2}0.03\left(3\cdot 10^{-3}\cdot 150\cdot\pi\right)^2\cdot 40.8 \cong 1.22 \ W$