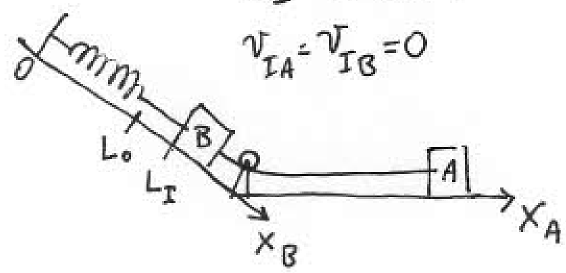
MID TERM N.2 SECTION 02



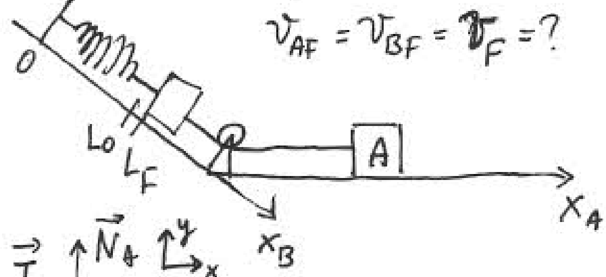
QUESTION 1

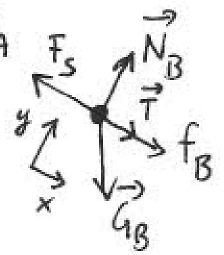
Since a speed has to worked out, one can apply the kinetic energy theorem.

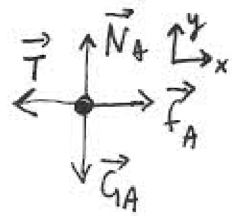
INITIAL $L_I = 0.65\ m$ $v_{IA} = v_{IB} = 0$




FINAL $L_F = 0.43\ cm$ $v_{AF} = v_{BF} = v_F = ?$




$$W = W_{F_S} + \underset{0}{\underset{\shortparallel}{W_{N_B}}} + W_{f_B} + \cancel{W_T} + W_{G_B} + \cancel{W_T} + \underset{0}{\underset{\shortparallel}{W_{N_A}}} + W_{f_A} + \underset{0}{\underset{\shortparallel}{W_{G_A}}} = W_{F_S} + W_{f_B} + W_{G_B} + W_{f_A} =$$

equal but opposite.

$$= \left[\frac{1}{2}k(L_I - L_0)^2 - \frac{1}{2}k(L_F - L_0)^2\right] - \mu_K m_B g \cos\theta \cdot (L_I - L_F) - m_B g \sin\theta (L_I - L_F) - \mu_K m_A g (L_I - L_F)$$

$$= \frac{1}{2}k\left[(L_I - L_0)^2 - (L_F - L_0)^2\right] - (L_I - L_F)\cdot g\left[\mu_K m_B \cos\theta + \mu_K m_A + m_B \sin\theta\right] =$$

$$= \frac{1}{2}500\left[(0.25)^2 - (0.03)^2\right] - 0.22\cdot 9.81\left[0.15\cdot 6\cdot \cos 12 + 0.15\cdot 2 + 6\cdot \sin 12\right] \cong 10.17\ J$$

the kinetic energy theorem reads $K_F - K_I = W \Rightarrow \frac{1}{2}(m_A + m_B)v_F^2 = 10.17$

$$v_F = \sqrt{\frac{2\cdot 10.17}{2+6}} \cong 1.59\ \frac{m}{s}$$

To answer to the following question one has to work out the accelerations $a_{Ax}$ and $a_{Bx}$ looking at the force diagrams above, not considering T:

$$a_{Bx} = \frac{-k\cdot\Delta L + m_B g \sin\theta + \mu_K m_B g \cos\theta}{m_B} \qquad a_{Ax} = \frac{\mu_K \cancel{m_A}\cdot g}{\cancel{m_A}}$$

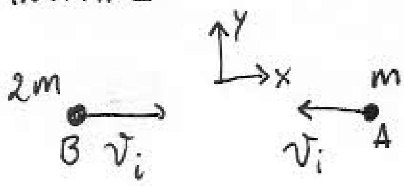
there is tension untill block B decelerate less than block A, i.e. $a_{Bx} \le a_{Ax}$

$$-\frac{k}{m_B}\Delta L + g\sin\theta + \mu_K g\cos\theta \le \mu_K g$$

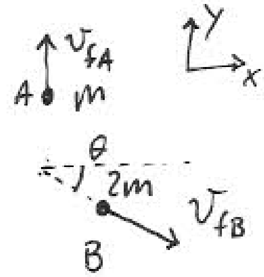
$$\Delta L \ge \frac{m_B g}{k}\left[\cancel{g}\sin\theta + \mu_K \cancel{g}(\cos\theta - 1)\right] = \frac{6\cdot 9.8}{500}\left[\sin 12^\circ + 0.15(\cos 12^\circ - 1)\right] \cong 2.4\ cm.$$

# QUESTION 2

INITIAL




FINAL




The momentum is preserved: $\vec{p}_i = \vec{p}_f$

$$\vec{p}_i = 2m\, v_i \cdot \hat{i} - m v_i \cdot \hat{i} = m\, v_i \cdot \hat{i}$$

$$\vec{p}_f = m\, v_{fA} \cdot \hat{j} + 2m\, v_{fB} \cdot \cos\theta \cdot \hat{i} - 2m\, v_{fB} \sin\theta\, \hat{j}$$

From the equivalence one finds:

$$\hat{i}: \quad \cancel{m} v_i = 2\cancel{m}\, v_{fB} \cdot \cos\theta$$

$$\hat{j}: \quad 0 = \cancel{m}\, v_{fA} - 2\cancel{m}\, v_{fB} \cdot \sin\theta$$

the 2 equations are not enough for the 3 ~~unknowns~~ unknowns $\theta, v_{fA}, v_{fB}$.

the third equation come from the conservation of kinetic energy (elastic collision): $K_i = K_f$

$$\cancel{\tfrac{1}{2}} 2\cancel{m} v_i^2 + \cancel{\tfrac{1}{2}}\cancel{m} v_i^2 = \cancel{\tfrac{1}{2}}\cancel{m}\, v_{fA}^2 + \cancel{\tfrac{1}{2}} 2\cancel{m}\, v_{fB}^2 \quad (*)$$

From $\hat{i}$, $v_{fB} = \dfrac{v_i}{2\cos\theta}$ From $\hat{j}$, $v_{fA} = v_i \dfrac{\sin\theta}{\cos\theta}$ Replacing them in $(*)$:

$$3\cancel{v_i^2} = \cancel{v_i^2} \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cancel{v_i^2}}{2\cos^2\theta} \rightarrow 6\cos^2\theta = 2\sin^2\theta + 1 \quad \text{i.e.:}$$

$$6(1-\sin^2\theta) = 2\sin^2\theta + 1 \rightarrow 8\sin^2\theta = 5 \rightarrow \theta = \sin^{-1}\left(\sqrt{\frac{5}{8}}\right) \simeq 52^\circ$$

$$v_{fB} = \frac{v_i}{2 \cdot \cos 52^\circ} = 0.812 \cdot v_i \qquad v_{fA} = v_i \tan 52^\circ = 1.28 \cdot v_i$$

QUESTION 3



Mechanical energy E is the sum of the kinetic energy K and the potential energy U :

$$E = K + U$$

E is preserved as long as all the forces have a potential U.

The kinetic energy theorem states that :

$$\Delta K = W$$

If all the forces have a potential then :

$$W = -\Delta U$$

Replacing W :

$$\Delta K = -\Delta U \rightarrow \Delta K + \Delta U = 0 \rightarrow \Delta(K+U) = 0 \rightarrow \Delta E = 0$$