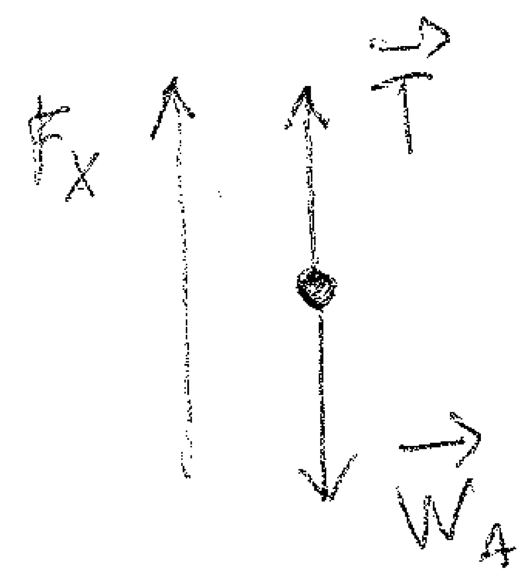
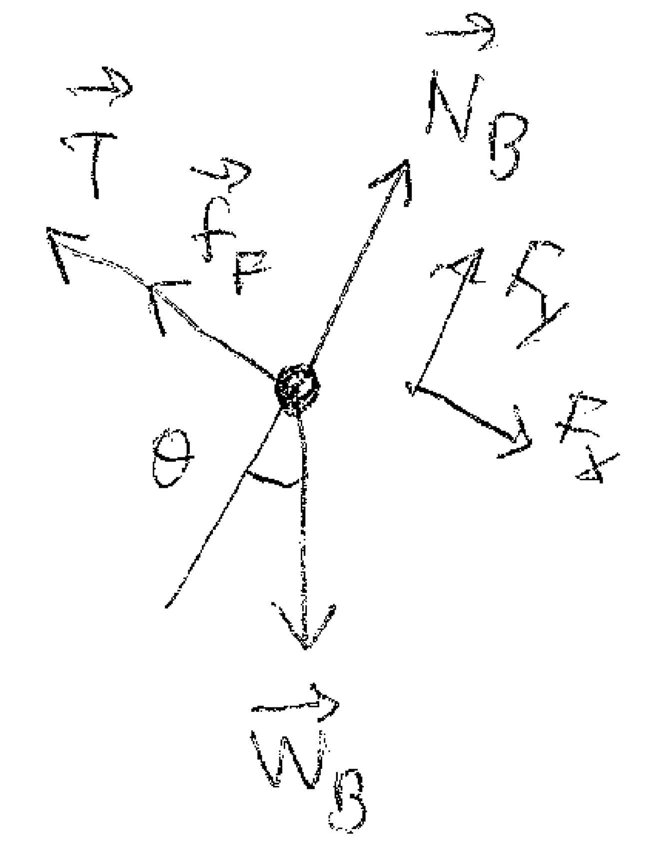
Question 1



$$\vec{a}_A = \frac{\vec{T} + \vec{W}_A}{m_A} = \frac{(T,0) + (-m_A g, 0)}{m_A}$$



$$\vec{a}_B = \frac{\vec{N}_B + \vec{W}_B + \vec{T} + \vec{f}_F}{m_B} = \frac{(0,N) + (m_B g \sin\theta, -m_B g \cos\theta) + (-T,0) + (-f_F,0)}{m_B}$$

$$a_{Ax} = \frac{T - m_A g}{m_A}$$

$$a_{Bx} = \frac{m_B g \sin\theta - T - f_F}{m_B}$$

$$a_{By} = \frac{-m_B g \cos\theta + N}{m_B} = 0 \Rightarrow N = m_B g \cos\theta$$

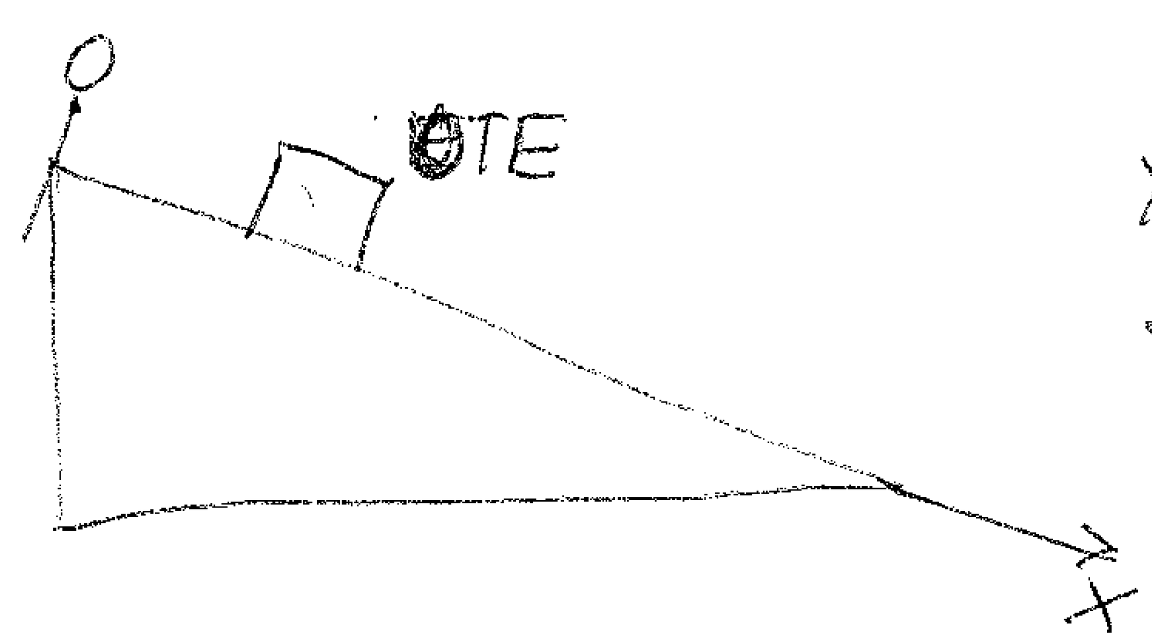
when the mass B is moving $f_F = f_{max} = \mu_k \cdot N = \mu_k \cdot m_B g \cos\theta$

the two accelerations are equal: $a_{Ax} = a_{Bx} = a$, hence the equations become:

$$\begin{cases} a = \dfrac{T - m_A g}{m_A} \Rightarrow T = m_A(a+g) \\[2ex] a = \dfrac{m_B g \sin\theta - T - \mu_k m_B g \cos\theta}{m_B} \end{cases} \Rightarrow a = \frac{m_B g \sin\theta - m_A g - \overbrace{\mu_k m_B g \cos\theta}^{f_F}}{m_A + m_B} \quad (*)$$

Let's replace the values: $a = \dfrac{6 \cdot 9.8 \cdot \sin 15^\circ - 2 \cdot 9.8 - 0.15 \cdot 6 \cdot 9.8 \cdot \cos 15^\circ}{2+6} = -1.61 \frac{m}{s^2}$

For the motion of block B we need a reference system:



$X(t) = X_0 + V_0 \cdot t + \frac{1}{2} a t^2 = 0.8 + 1.5 \cdot t - 0.805 \cdot t^2$

The system (blocks A and B) stops the motion because the acceleration is opposite to the initial ~~spee~~ velocity.

$v = 1.5 - 1.61 \cdot t = 0 \Rightarrow t = \frac{1.5}{1.61} = 0.932\,s$

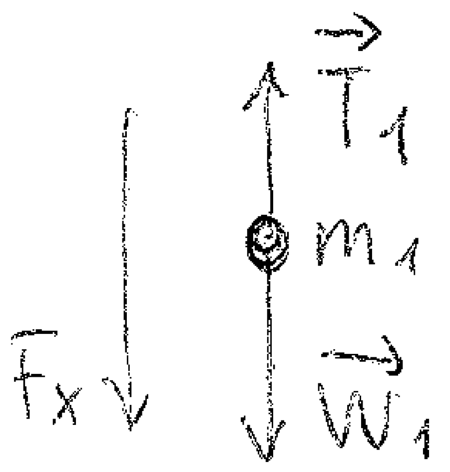
the position of B when its speed is 0 is then $X = 0.8 + 1.5 \cdot 0.932 - 0.805 \cdot 0.932^2 = 1.5\,m$

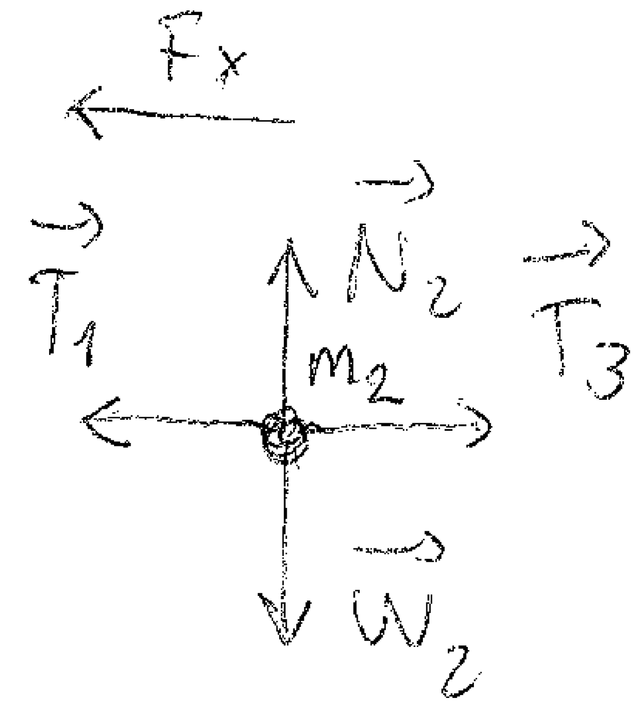
From eq. (*) let's find the friction that makes $a = 0$. The friction force results:

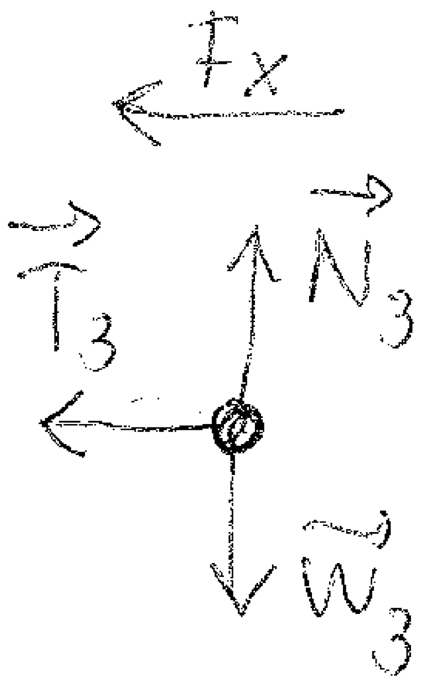
$f_F = m_B \cdot g \sin\theta - m_A \cdot g = 6 \cdot 9.8 \cdot \sin 15^\circ - 2 \cdot 9.8 = -4.38\,N$ We have to compare this value with the maximum friction force $\mu_s \cdot m_B g \cdot \cos\theta = 0.3 \cdot 6 \cdot 9.8 \cdot \cos 15^\circ = 17.0\,N$

Hence, there is enough friction to hold the system at rest.

Question 2





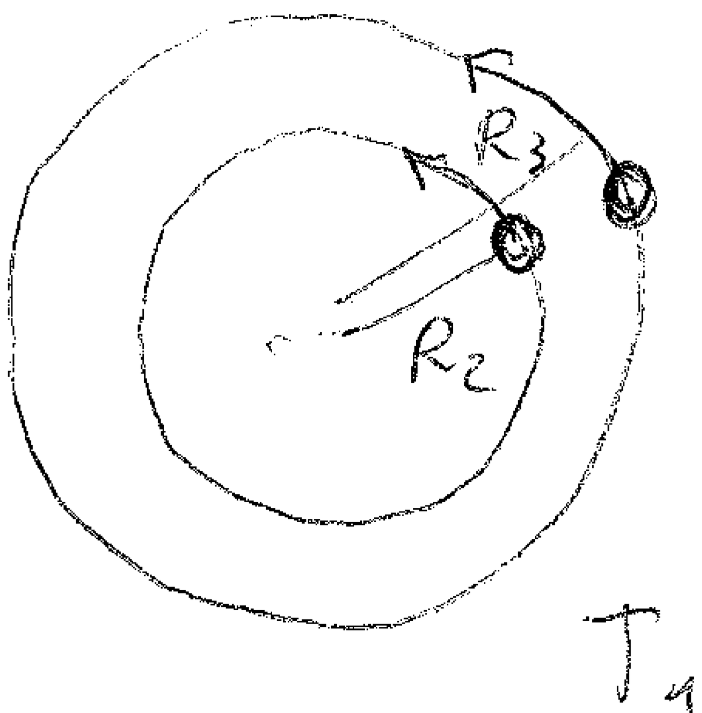


$$a_{1x} = \frac{m_1 g - T_1}{m_1} = 0 \qquad a_{2x} = \frac{T_1 - T_3}{m_2} = \frac{v_2^2}{R_2} \qquad a_{3x} = \frac{T_3}{m_3} = \frac{v_3^2}{R_3}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$T_1 = m_1 g = 0.09 \cdot 9.8 = 0.882\,N \qquad T_3 = T_1 - m_2 \frac{v_2^2}{R_2} \qquad T_3 = m_3 \frac{v_3^2}{R_3} \; (**)$$

Hence, $T_1 - m_2 \frac{v_2^2}{R_2} = m_3 \frac{v_3^2}{R_3}$ (*) / v_2 e v_3 are not indipendent:



$$v_2 = \frac{2\pi R_2}{t} \text{ and } v_3 = \frac{2\pi R_3}{t} \Rightarrow \frac{v_3}{v_2} = \frac{R_3}{R_2} \Rightarrow v_3 = v_2 \frac{R_3}{R_2}$$

same time

Equation (*) ~~de~~ becomes:

$$T_1 - m_2 \frac{v_2^2}{R_2} = \frac{m_3}{\cancel{R_3}} \frac{R_3^{\cancel{2}}}{R_2^2} \cdot v_2^2 \Rightarrow v_2 = \sqrt{\frac{T_1}{\frac{m_2}{R_2} + m_3 \frac{R_3^2}{R_2}}} = \sqrt{\frac{0.882}{\frac{0.12}{0.7} + 0.06 \frac{0.9^2}{0.7}}} =$$

$$= 1.91 \; \frac{m}{s}$$

$$\downarrow$$

$$v_3 = \frac{R_3}{R_1} \cdot v_2 = \frac{0.9}{0.7} 1.91 = 2.46 \; \frac{m}{s}$$

From equation (**):

$$T_3 = 0.06 \cdot \frac{2.46^2}{0.9} = 0.403\,N$$

When the string is ~~cut~~ cut between m_2 and m_3, the latter mass has not any radial force and hence it ~~s~~ moves on a straight line tangencial to the circonference and $T_3 = 0$. the equations for m_1 and ~~are~~ m_2 are:

$$a = \frac{m_1 g - T_1}{m_1} \qquad a + \frac{v_2^2}{R_2} = \frac{T_1}{m_2} \Rightarrow$$

$$\Rightarrow \begin{cases} T_1 = m_1 (g - a) \\ T_1 = m_2 \left(a + \frac{v_2^2}{R_2}\right) \end{cases} \Rightarrow \quad m_2\left(a + \frac{v_2^2}{R_2}\right) = m_1 (g - a)$$

$$\downarrow$$

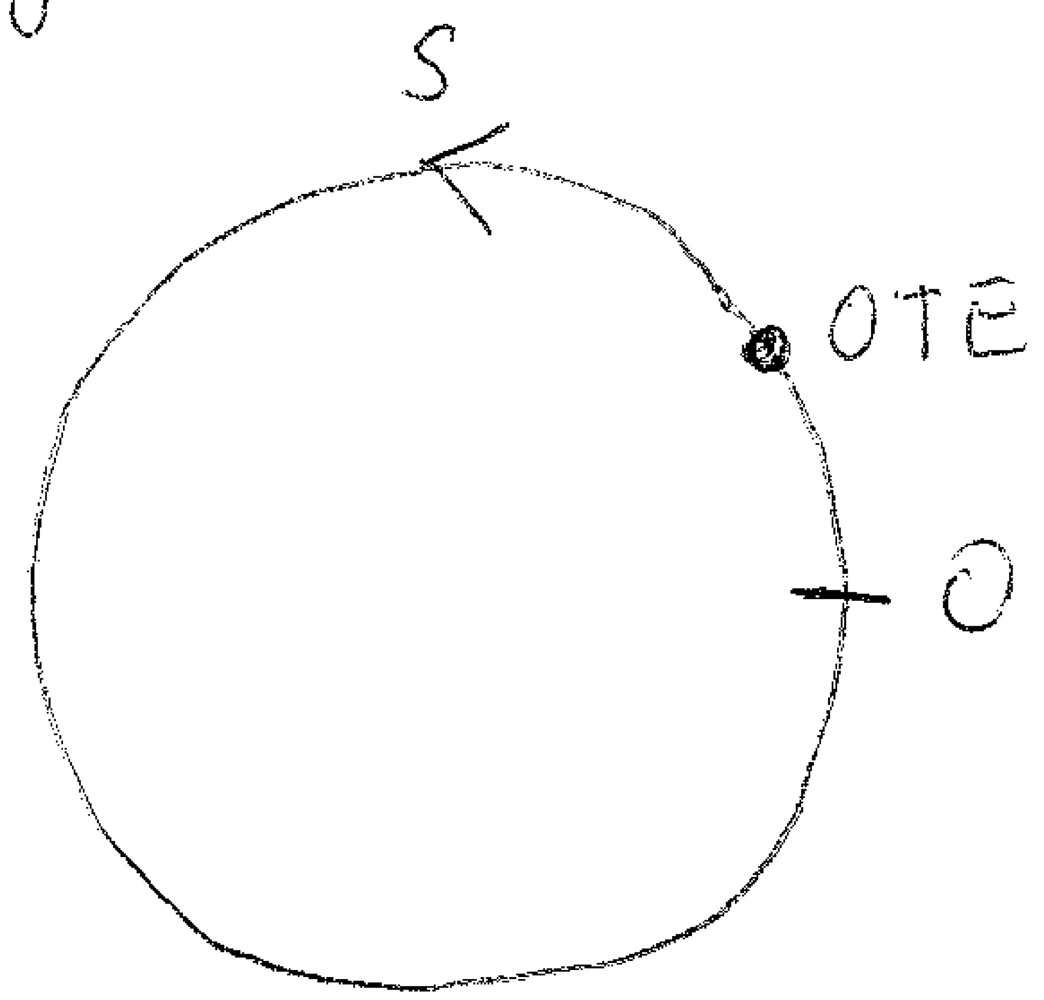
$$a = \frac{m_1 g - m_2 \frac{v_2^2}{R_2}}{m_1 + m_2} =$$

$$= \frac{0.09 \cdot 9.81 - 0.12 \frac{1.91^2}{0.7}}{0.09 + 0.12} = 1.23 \; \frac{m}{s^2}$$

m_2 starts to fall to the center.

Question 3

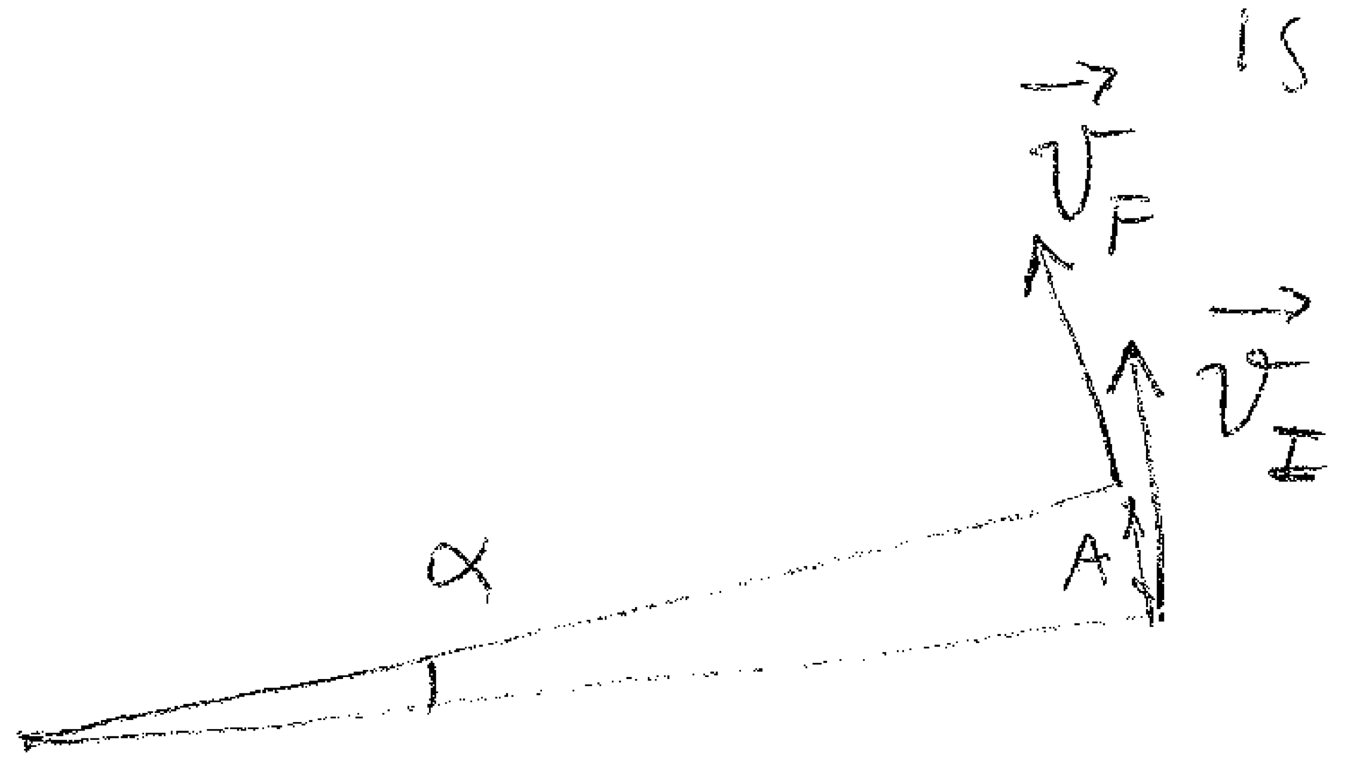
To describe the motion at first we have to set a reference system. For a circular trajectory a reference system can be:

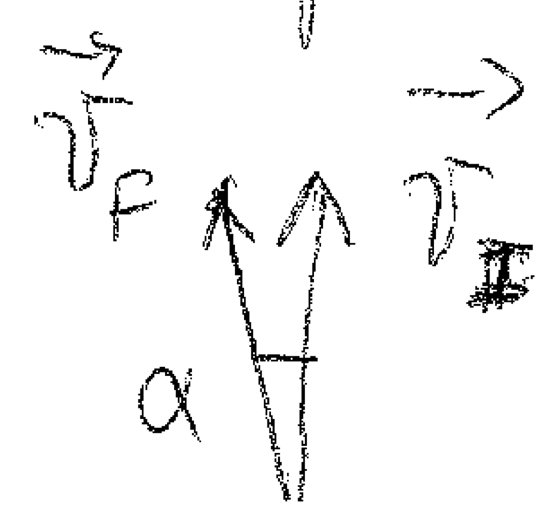


The zero time event is the event we consider as initial. The motion is then:

$$S(t) = S_0 + v_0 \cdot t$$

S_0 is the ~~tim~~ space at time zero and v_0 is the speed of the object at time zero.

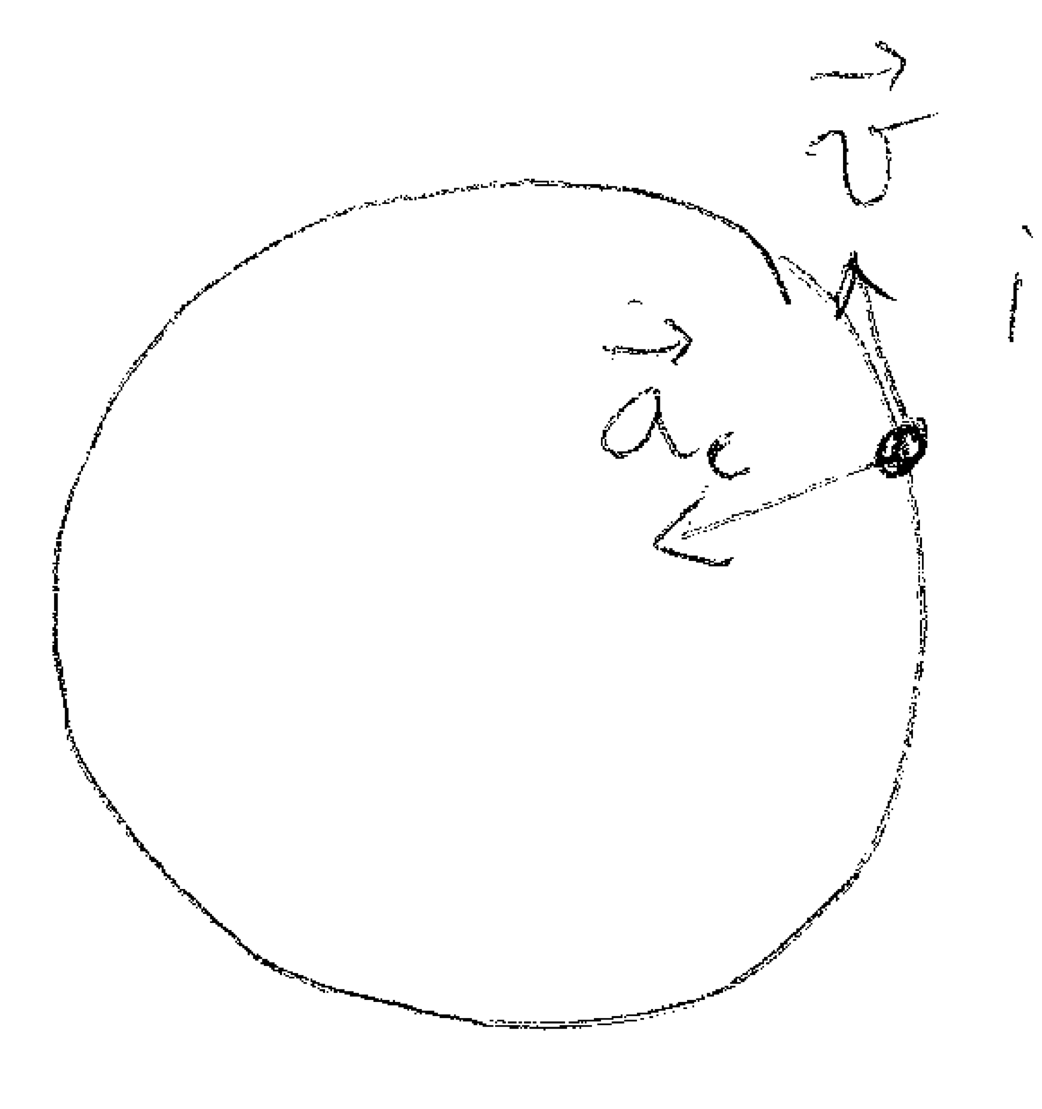




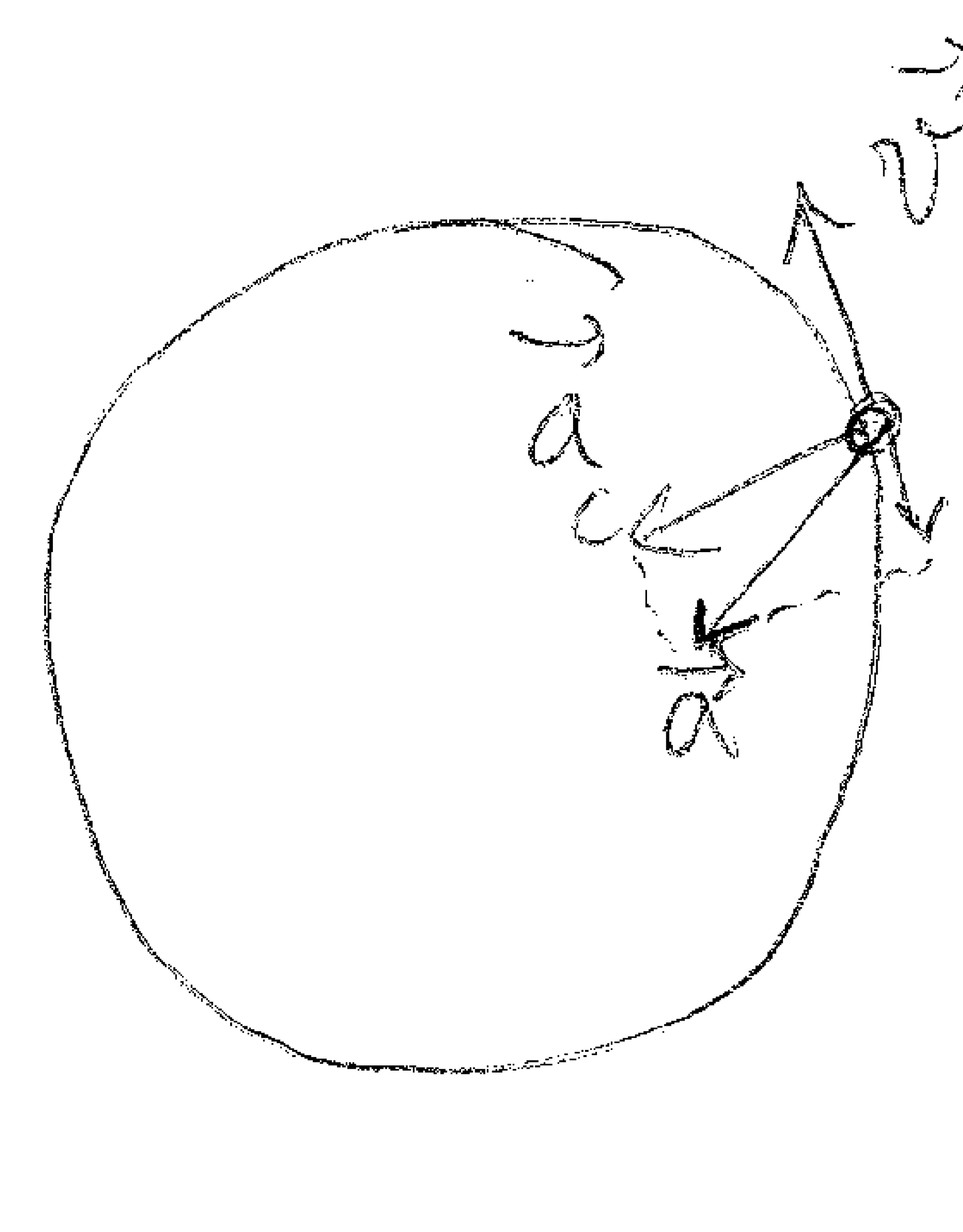
$$|\vec{a}| = \frac{|\vec{v}_F - \vec{v}_I|}{\Delta t} \simeq \frac{|\vec{v}| \cdot \alpha}{\Delta t} =$$

$$= |\vec{v}| \cdot \frac{A/R}{\Delta t} = |\vec{v}| \cdot \frac{A}{\Delta t} \cdot \frac{1}{R} =$$

$$= |\vec{v}| \cdot |\vec{v}| \frac{1}{R} = \frac{v^2}{R}$$



if the motion is at constant speed.



if the motion is decelerating.