The lowest order expansion of GW

$$h^{TT}_{ij}(t,\vec{x}) = \frac{1}{r}\frac{4G}{c^4}\Lambda_{ij,kl}\cdot S^{kl}\left(t-\frac{r}{c}\right)$$

The indeces of h^{TT} are latin because we can always find $4\,x^{\nu}$ that make $h_{0i}=0$ and consequently $\partial^0 h_{00}=0$ through the Lorentz condition. Hence, h_{00} is constant and we can choose it $=0$ because we want a GW and not a feeble DC component of the gravitational field.

The matrix S^{kl} is:

$$S^{kl} = \int d^3x\, T^{kl}\left(t-\frac{r}{c},\vec{x}\right)$$

We will demonstrate here that in order to calculate S^{kl} we need to know just the component T^{00} and not all the 6 independent components T^{kl}.

The demonstration is based on the conservation law of the energy-~~impu~~momentum tensor:

$$\partial_\mu T^{\mu\nu}=0 \rightarrow \partial_0 T^{0\nu}+\partial_i T^{i\nu}=0 \rightarrow \partial_0 T^{0\nu} = -\partial_i T^{i\nu}$$

Let's procede. We observe that:

$$S^{kl} = \int d^3x\, T^{kp}\delta_p{}^{l} = \int d^3x\, T^{kp}\partial_p x^l = \text{we integrate by parts} =$$

$$= \int d^3x\, \partial_p\left(T^{kp}\cdot x^l\right) - \int d^3x\, \partial_p T^{kp}\cdot x^l = \text{divergence theorem} =$$

$$= \underbrace{\int d^2x\, \hat{n}_p\left(T^{kp}\cdot x^l\right)}_{\substack{\text{Surface integral.}\\ T^{kp}\text{ is zero outside}\\ \text{the source}}} + \int d^3x\, \partial_0 T^{k0}\cdot x^l = \frac{1}{c}\int d^3x\, \dot{T}^{k0}\cdot x^l =$$

$$= \dot{P}^{k,l}$$

We see that there is a problem of symmetry.
While $S^{kl} = S^{lk}$ because $T^{\mu\nu}$ is simmetric, the same thing cannot be said to $P^{k,l}$: $P^{k,l} \neq P^{l,k}$
On the other hand we observe that we could have made a different choice at the beginning, i.e.:

$$S^{kl} = \int d^3x\, T^{pl} \delta_p{}^k = \int d^3x\, T^{pl} \partial_p x^k = \ldots = \int d^3x\, \partial_0 T^{0l} \cdot x^k =$$

$$= \frac{1}{c} \int d^3x\, \dot{T}^{0l} \cdot x^k = \dot{P}^{l,k}$$

Therefore, in order to preserve the simmetry explicitly, one could write S^{kl} in the following form:

$$S^{kl} = \int d^3x\, \frac{1}{2} \left(T^{pl} \delta_p{}^k + T^{kp} \delta_p{}^l \right) = \ldots = \frac{1}{2} \left(\dot{P}^{k,l} + \dot{P}^{l,k} \right)$$

We can continue to apply the same tric to $P^{k,l}$ and $P^{l,k}$:

$$P^{k,l} = \frac{1}{c} \int d^3x\, T^{0k} \cdot x^l = \frac{1}{c} \int d^3x\, T^{0p} \cdot \delta_p{}^k \cdot x^l = \frac{1}{c} \int d^3x\, T^{0p} \cdot \partial_p x^k \cdot x^l =$$

$$= \underbrace{\frac{1}{c} \int d^2x\, n_p \left[T^{0p} \cdot x^k \cdot x^l \right]}_{=0} - \frac{1}{c} \int d^3x\, \partial_p \left[T^{0p} \cdot x^l \right] x^k = \frac{1}{c} \int d^3x\, \partial_0 T^{00} x^k x^l +$$

$$- \frac{1}{c} \int d^3x\, T^{0p} \cdot \delta_p{}^l \cdot x^k = \frac{1}{c} \int d^3x\, \partial_0 T^{00} \cdot x^k \cdot x^l - P^{l,k}$$

So: $P^{k,l} + P^{l,k} = \frac{1}{c^2} \int d^3x\, \dot{T}^{00} \cdot x^k \cdot x^l$ Finally:

$$S^{kl} = \frac{1}{2} \left(\dot{P}^{k,l} + \dot{P}^{l,k} \right) = \frac{1}{2c^2} \int d^3x\, \ddot{T}^{00} \cdot x^k \cdot x^l = \frac{1}{2} \ddot{\mathcal{M}}^{kl}$$