POWER EMITTED BY A QUADRUPOLAR GW

$$t_{00} = \frac{c^2}{16\pi G} \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \quad (1)$$

As for any wave the infinitesimal ~~intensity~~ power dP is:

$$dP = dA \cdot \varepsilon \cdot v \quad (2)$$

where dA is the infinitesimal surface, ε is the energy density of the wave and v its speed.

$[dP] = m^2 \frac{J}{m^3} \cdot \frac{m}{s} = W$ ok!

The intensity is $I = \varepsilon \cdot v$.

Taking a sphere of radius r from the source, from (2) we have

$$P = \int t_{00} \cdot c \; r^2 d\Omega \longrightarrow \frac{dP}{d\Omega} = r^2 \frac{c^3}{16\pi G} \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \; (3)$$

For a binary:

$$h_+ = \frac{1}{r} \frac{4G\mu\omega^2 R^2}{c^4} \frac{1+\cos^2\theta}{2} \cos(2\omega t_{ret} + 2\varphi) \quad (4)$$

$$h_\times = \frac{1}{r} \frac{4G\mu\omega^2 R^2}{c^4} \cos\theta \sin(2\omega t_{ret} + 2\varphi) \quad (5)$$

Considering that $\langle \dot{\cos}^2 \rangle = \langle \dot{\sin}^2 \rangle = 4\omega^2 \cdot \frac{1}{2} = 2\omega^2$ (6)

we have:

$$\frac{dP}{d\Omega} = \frac{r^2c^3}{16\pi G}\frac{1}{r^2}\frac{16G^2M^2\omega^4R^4}{c^8}2\omega^2 g(\theta) =$$

$$= \frac{2G}{\pi}\frac{M^2R^4\omega^6}{c^5}g(\theta) \quad (7) \quad \text{where } g(\theta) = \left[\cos^2\theta + \frac{(1+\cos^2\theta)^2}{4}\right] =$$

$$= \cos^2\theta + \cos^2 2\theta \quad (8)$$

$$\int g(\theta)\,d\Omega = \int g(\theta)\sin\theta\, d\varphi\, d\theta = 2\pi\int_0^{\pi} g(\theta)\sin\theta\, d\theta =$$

$$= 2\pi\int_1^{-1} g(\cos\theta)\,d(-\cos\theta) = 2\pi\int_{-1}^{1}\left[x^2 + \frac{(1+x^2)^2}{4}\right]dx =$$

$$= 2\pi\int_{-1}^{1}\left[x^2 + \frac{1+2x^2+x^4}{4}\right]dx = 2\pi\int_{-1}^{1}\left[\frac{1}{4} + \frac{3}{2}x^2 + \frac{x^4}{4}\right]dx =$$

$$= 2\pi\left[\frac{x}{4} + \frac{x^3}{2} + \frac{x^5}{20}\right]_{-1}^{1} = 4\pi\,\frac{5+10+1}{20} = \frac{16\pi}{5} \quad (9)$$

Therefore, the total power radiated is:

$$P = \int \frac{dP}{d\Omega}\,d\Omega = \frac{32}{5}G\,\frac{M^2R^4\omega^6}{c^5} \quad (10)$$

$\boxed{\omega \text{ is that of the orbit}}$

since $v = R\cdot\omega$ $\boxed{v \text{ is the speed of one mass as seen from the other}}$

$$P = \frac{32}{5}G\frac{M^2}{R}\left(\frac{v}{c}\right)^5\cdot\omega \longrightarrow \int_0^T P\,dt = \langle E_{emitted}\rangle_T = \frac{64\pi}{5}G\frac{M^2}{R}\left(\frac{v}{c}\right)^5$$

(11)