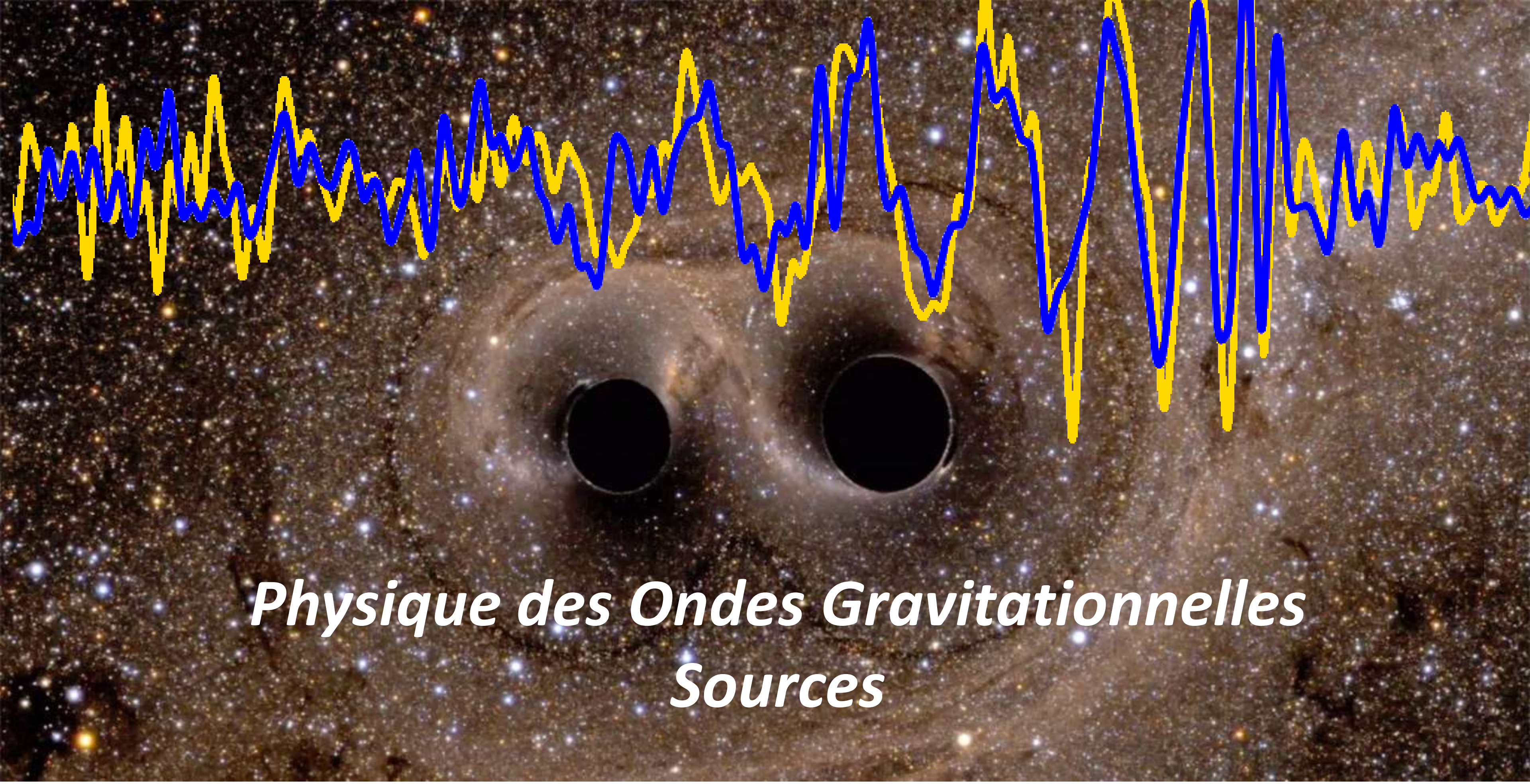

Physique des Ondes Gravitationnelles
Sources

iLM
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Lyon 1

La masse réduite μ

- La Lagrangien de 2 corps

$$\mathcal{L} = \frac{1}{2}\, m_1 \dot{\vec{r_1}}^2 + \frac{1}{2}\, m_2 \dot{\vec{r_2}}^2 - \mathrm{V}(|\vec{r_1} - \vec{r_1}|)$$

- Système de repaire du c.d.m.

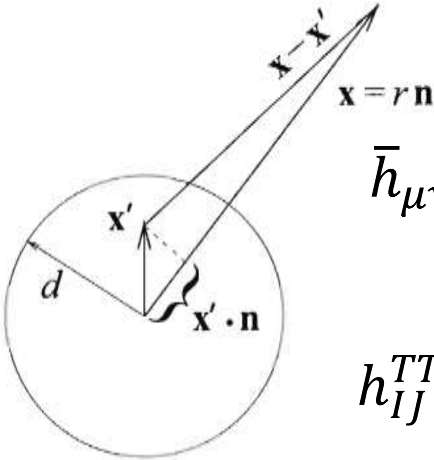
$$m_1\vec{r_1} + m_2\vec{r_2} = \vec{0} \qquad \vec{r_1} - \vec{r_2} = \vec{r}$$

$$\mathcal{L} = \frac{1}{2}\, \mu \dot{\vec{r}}^2 - \mathrm{V}(|\vec{r}|)$$

$$\mu = \frac{m_1 \cdot m_2}{m_1 + m_2} \qquad M = m_1 + m_2 \quad \mathrm{V}(|\vec{r}|) = -G\frac{M \cdot \mu}{r}$$

MM)

L'expansion faible vitesse



$$\bar{h}_{\mu\nu}(t,\vec{x}\,) = \frac{4G}{c^4}\int d^3x' \frac{1}{\left|\vec{x}-\overrightarrow{x'}\right|}\cdot T_{\mu\nu}\left(t-\frac{\left|\vec{x}-\overrightarrow{x'}\right|}{c},\overrightarrow{x'}\right)$$

$$h_{IJ}^{TT}(t,\vec{x}) = \frac{1}{r}\frac{4G}{c^4}\Lambda_{ij,kl}(\hat{n})\int d^3x' \;\; T_{\mu\nu}\left(t-\frac{r}{c}+\frac{\overrightarrow{x'}\cdot\hat{n}}{c},\overrightarrow{x'}\right)$$

et donc on arrive au résultat suivant

$$h_{IJ}^{TT}(t,\vec{x}) = \frac{1}{r}\frac{4G}{c^4}\Lambda_{ij,kl}(\hat{n})\left[S^{kl}(t_{\text{ret}})+\frac{1}{c}n_m\dot{S}^{m,kl}(t_{\text{ret}})+\frac{1}{c^2}n_m n_p\ddot{S}^{mp,kl}(t_{\text{ret}})+\cdots\right]$$

$$S^{kl} = \frac{1}{2}\ddot{\mathcal{M}}^{kl} \qquad \mathcal{M}^{kl} = \frac{1}{c^2}\int d^3x \;\; T^{00}\cdot x^k x^l$$

MM)

La radiation du quadripôle Q de masse

$$\left[h_{IJ}^{TT}(t,\vec{x})\right]_{\text{quad}} = \frac{1}{r}\frac{2G}{c^4} \cdot \Lambda_{ij,kl}(\hat{n}) \cdot \ddot{Q}^{kl}(t - r/c)$$

$$Q^{kl} = \mathcal{M}^{kl} - \frac{1}{3}\delta^{kl}\mathcal{M}^{ii}$$

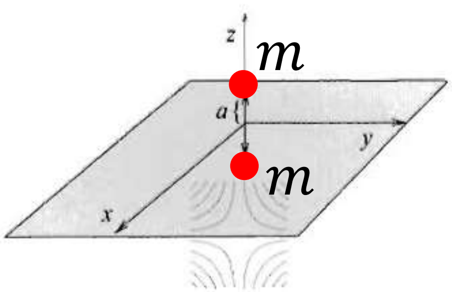
$$\mathcal{M}^{kl} = \int d^3x \;\; \rho(t,\vec{x})\, x^k\, x^l$$

- Pour une radiation que se propage selon z

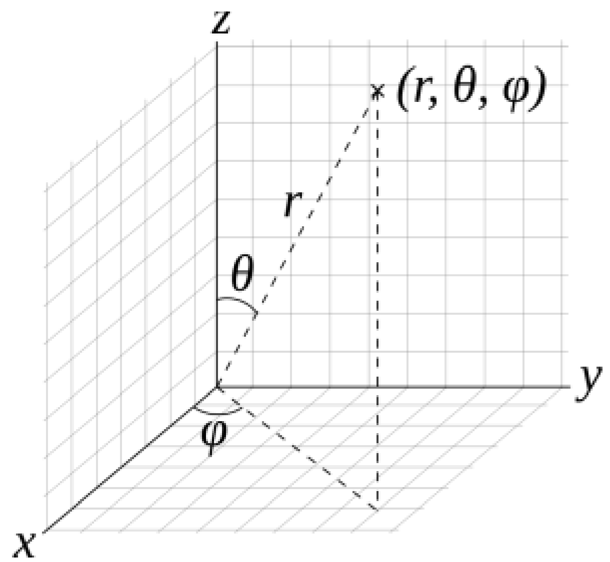
$$h_+ = \frac{1}{r}\frac{G}{c^4}\left(\ddot{Q}^{11} - Q^{22}\right)$$
$$h_\times = \frac{2}{r}\frac{G}{c^4}\ddot{Q}^{12}$$

MM)

OG produit par deux masses oscillantes



$$z_0(t) = a \cdot \cos \omega_s t$$

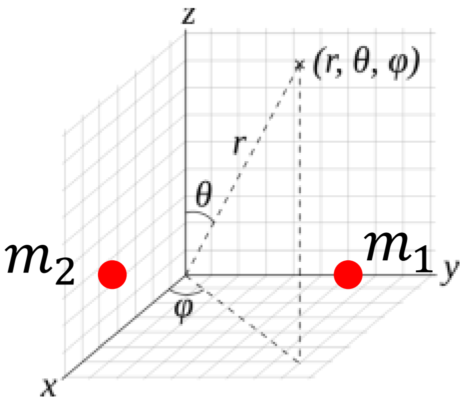


$$h_+(t,\theta,\varphi) = \frac{1}{r}\frac{8\,G\,m\,a^2\,{\omega_s}^2}{c^4} \cdot \sin^2\theta \cdot \cos 2\varphi \cdot \cos(2\omega_s \cdot t_{\text{ret}})$$

$$h_\times(t,\theta,\varphi) = \frac{1}{r}\frac{8\,G\,m\,a^2\,{\omega_s}^2}{c^4} \cdot \sin^2\theta \cdot \sin 2\varphi \cdot \cos(2\omega_s \cdot t_{\text{ret}})$$

MM)

OG produit par deux masses en orbite circulaire



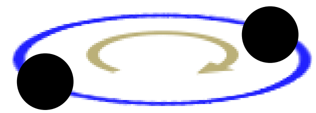
$$x_0(t) = R \cdot \cos\left(\omega_s t + \frac{\pi}{2}\right)$$
$$y_0(t) = R \cdot \sin\left(\omega_s t + \frac{\pi}{2}\right)$$
$$z_0(t) = 0$$

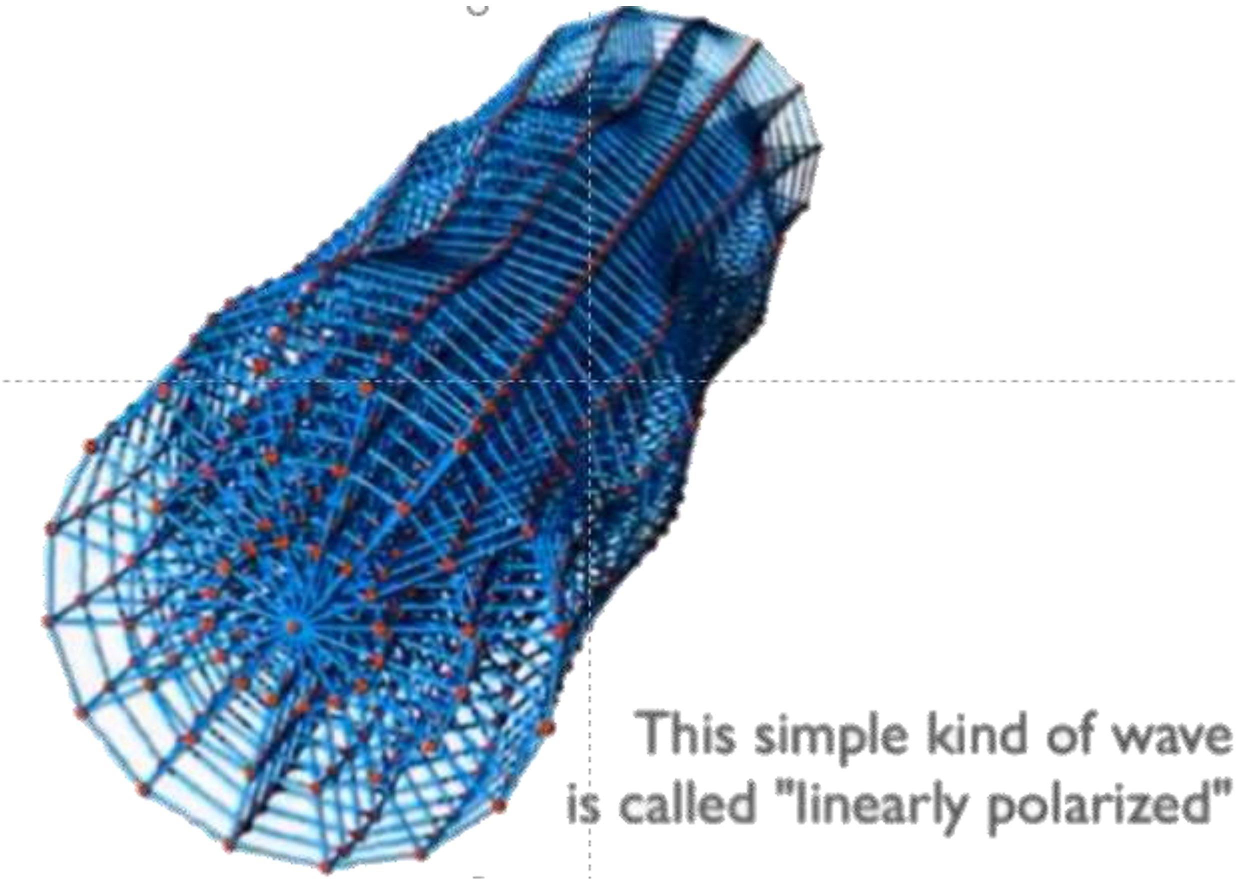
$$h_+(t, \theta, \varphi) = \frac{1}{r} \frac{4\, G\, \mu\, R^2\, {\omega_s}^2}{c^4} \cdot \left(\frac{1 + \cos^2\theta}{2}\right) \cdot \cos(2\omega_s \cdot t_{\text{ret}} + 2\varphi)$$
$$h_\times(t, \theta, \varphi) = \frac{1}{r} \frac{4\, G\, \mu\, R^2\, {\omega_s}^2}{c^4} \cdot \cos\theta \cdot \sin(2\omega_s \cdot t_{\text{ret}} + 2\varphi)$$

MM)

2 ou 3 trucs sur les OG: Polarisation linéaire

$\theta = \pi/2$

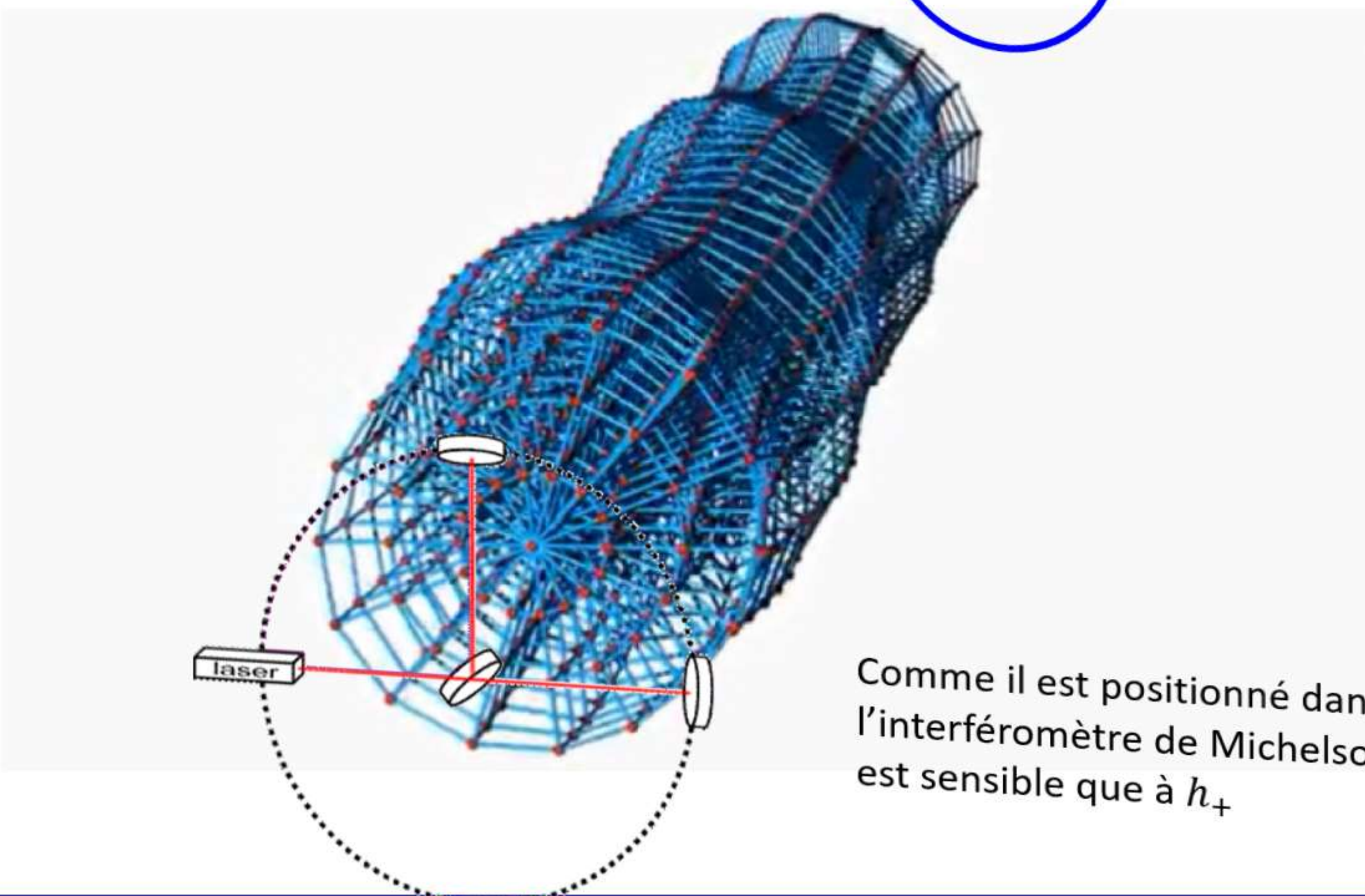




9)

2 ou 3 trucs sur les OG: Polarisation circulaire

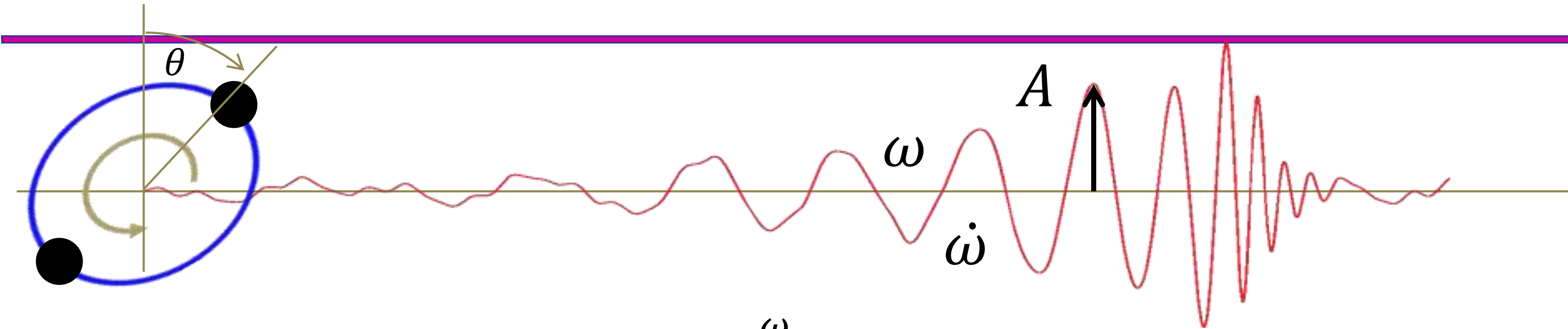
$$\theta = 0$$



Comme il est positionné dans cette figure l'interféromètre de Michelson est sensible que à h_+

9)

L’information de la binaire contenu dans l’OG



- Fréquence orbitale $\omega_{\text{orb}} = \frac{\omega}{2}$

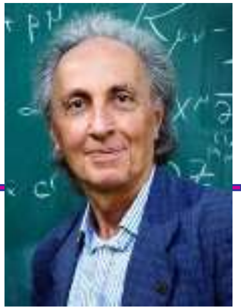
Malheureusement nous n’avons pas les 2 polarisations

- Inclination orbitale θ: $\frac{h_+}{h_\times} = \frac{1+\cos^2\theta}{2\ \cos\theta}$
- La distance d: $\mathrm{A} = A(\omega, \text{masses}, d, \theta)$ $A = \frac{4}{c^4}\omega^{\frac{2}{3}} \cdot (G\ \mathcal{M}_c)^{\frac{5}{3}} \cdot \frac{1}{d}\begin{cases} 1+\cos^2\theta \\ 2\ \cos\theta \end{cases}$
- Les masses m_1 et m_2

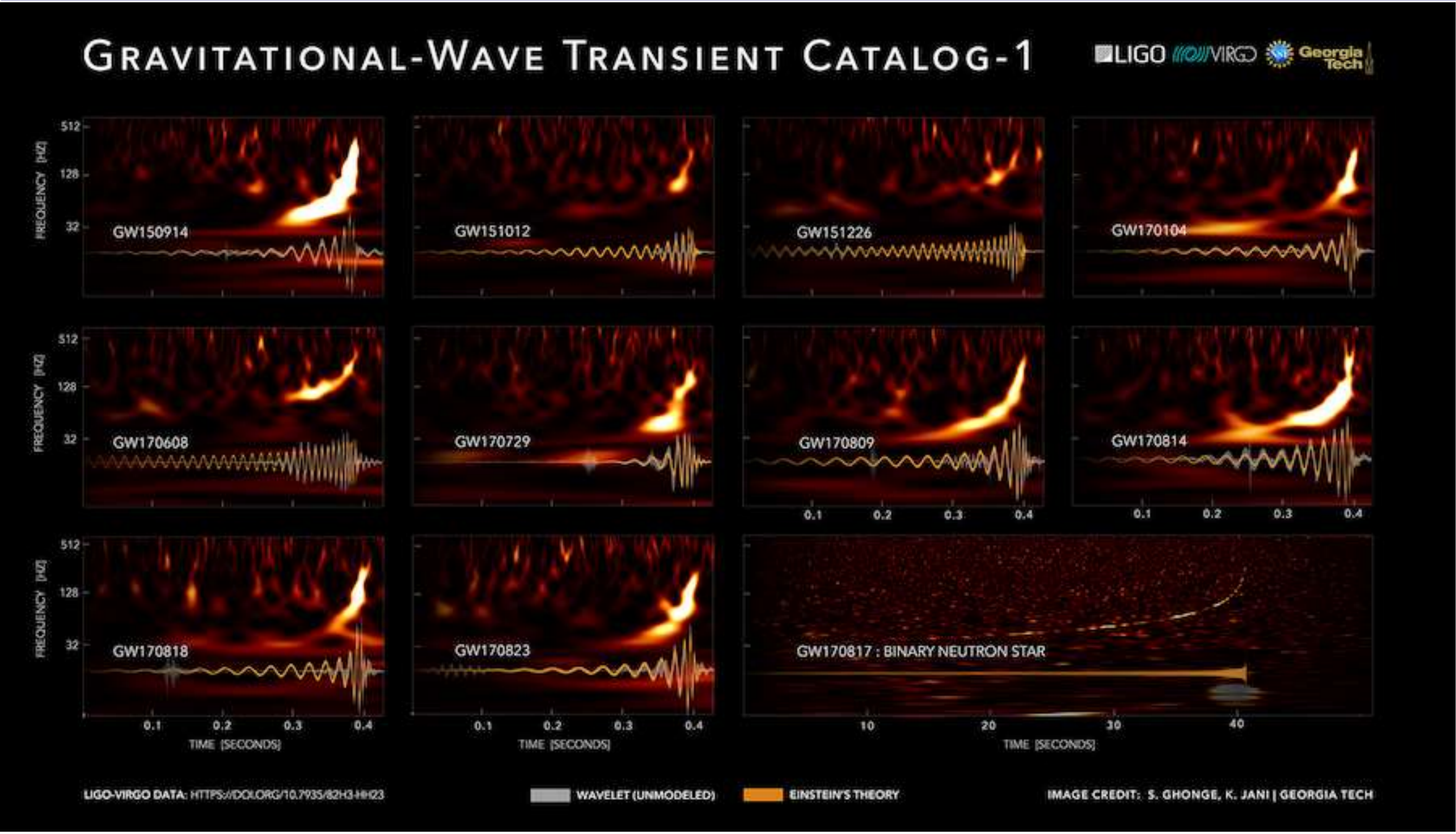
– Chirp mass $\mathcal{M}_c$: $\mathcal{M}_c^{\frac{5}{3}} = \mu \cdot M^{\frac{2}{3}} = m_1 \cdot m_2/(m_1+m_2)^{\frac{1}{3}}$

– $\frac{\dot{\omega}}{\omega^{\frac{11}{3}}} = \frac{12\sqrt[3]{2}}{5c^5} \cdot (G\ \mathcal{M}_c)^{\frac{5}{3}}$

L. Blanchet, T. Damour pardonnez-moi !!!



Les différentes formes des signaux



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