Momenta

Momenta of the energy density T^{00}

$$\mathcal{M} = \frac{1}{c^2}\int d^3x \, T^{00}(t,\vec{x}) \qquad (1)$$

$$\mathcal{M}^i = \frac{1}{c^2}\int d^3x \, x^i \, T^{00}(t,\vec{x}) \qquad (2)$$

$$\mathcal{M}^{ij} = \frac{1}{c^2}\int d^3x \, x^i x^j \, T^{00}(t,\vec{x}) \qquad (3)$$

Momenta of the momentum density $\frac{T^{0i}}{c}$

$$\mathcal{P}^i = \frac{1}{c}\int d^3x \, T^{0i}(t,\vec{x}) \qquad (4)$$

$$\mathcal{P}^{ij} = \frac{1}{c}\int d^3x \, x^j . T^{0i}(t,\vec{x}) \qquad (5)$$

$$\mathcal{P}^{ijk} = \frac{1}{c}\int d^3x \, x^j x^k . T^{0i}(t,\vec{x}) \qquad (6)$$

From the conservation law of $T^{\mu\nu}$:

$$\partial_\nu T^{0\nu} = 0$$

follow important relations between $\mathcal{M}$ and $\mathcal{P}$

$$\partial_0 T^{00} = -\partial_i T^{0i}$$

$$\downarrow$$

$$\frac{1}{c}\int_V d^3x\, \dot{T}^{00} = -\int_V d^3x\, \partial_i T^{0i} = -\int_S d^2x\, \hat{n}\cdot\overrightarrow{T^{0i}} = 0$$

because the surface S is outside the source

$$\downarrow$$

$$\dot{\mathcal{M}} = 0 \quad (7)$$

This resulto seems trivial but in reality is completely false!!! When a source radiate $\dot{\mathcal{M}}<0$ but here, since we work with flat metrics, the GW do not carry away energy .... $\boxed{\text{BACK ACTION BRAKES } \dot{\mathcal{M}}=0}$

$$\dot{\mathcal{M}}^i = \frac{1}{c^2}\cdot c\int d^3x\, x^i \partial_0 T^{00} = -\frac{1}{c}\int d^3x\, x^i \partial_j T^{0j} =$$

$$= -\frac{1}{c}\left\{\int d^3x\, \partial_j(x^i\cdot T^{0j}) - \int d^3x\, \delta^i_j T^{0j}\right\} =$$

$$= -\frac{1}{c}\left\{\int_S d^2x\, n_j x^i T^{0j} - c\mathcal{P}^i\right\} = \mathcal{P}^i \quad (8)$$

Again, S goes over points where $T^{0j}=0$.

Similarly:

$$\dot{\mathcal{M}}^{ij} = \mathcal{P}^{i,j} + \mathcal{P}^{j,i} \quad (9)$$

$$\dot{\mathcal{M}}^{ijk} = \mathcal{P}^{i,jk} + \mathcal{P}^{j,ki} + \mathcal{P}^{k,ij} \quad (10)$$

$$\dot{\mathcal{P}}^{i} = \frac{1}{c}\int d^3x\, \partial_t T^{0i} = \int d^3x\, \partial_0 T^{0i} = -\int d^3x\, \partial_j T^{ji} =$$

$$= -\int_S d^2x\, n_j T^{ji} = 0 \qquad (11)$$

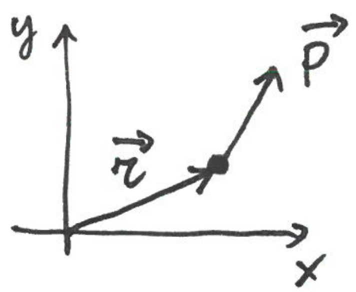
$$\dot{\mathcal{P}}^{i,j} = \int d^3x\, \partial_0 T^{0i}\cdot x^j = -\int d^3x\, \partial_k T^{ki}\cdot x^j =$$

$$= -\int d^3x\, \partial_k \left(T^{ki}\cdot x^j\right) + \int d^3x\, T^{ki}\cdot \delta^j_k =$$

$$= 0 + \int d^3x\, T^{ij} = S^{ij} \qquad (12)$$

$$\dot{\mathcal{P}}^{i,jk} = S^{ij,k} + S^{ik,j} \qquad (13)$$

In 2-D :



the angular momentum (along z) is

$L_z = x\cdot p_y - y\cdot p_x$

in 3-D $L_i = x_j p_k - x_k p_j$

So, the total angular momentum derivative is $\dot{\mathcal{P}}^{ij} - \dot{\mathcal{P}}^{j,i} \Rightarrow S^{ij} - S^{ji} = 0$

$\dot{\mathcal{M}} = 0$ conservation of mass

$\dot{\mathcal{P}}^{i} = 0$ conservation of momentum

$\dot{\mathcal{P}}^{ij} - \dot{\mathcal{P}}^{j,i} = 0$ conservation of angular momentum