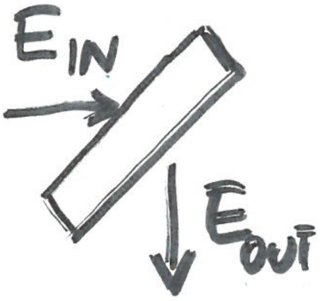
The modulation of light by GW.

The Michelson interferometer allows the GW modulating the laser beam. Let's analyse this modulation.



$$E_{IN} = E_0 e^{-i\omega t} \qquad (1)$$

$$E_{OUT} = E_{OUT}^{(x)} - E_{OUT}^{(y)} \qquad (2)$$

$$\begin{cases} E_{OUT}^{(x)} = \frac{E_0}{2} e^{-i\omega\left(t-\frac{2L_x}{c}\right)} e^{i\delta\varphi} & (3) \\ E_{OUT}^{(y)} = +\frac{E_0}{2} e^{-i\omega\left(t-\frac{2L_y}{c}\right)} e^{-i\delta\varphi} & (4) \end{cases}$$

where

$$\delta\varphi = \underbrace{\frac{\omega}{\Omega} \sin\left(\Omega \frac{L}{c}\right)}_{2\pi \frac{L}{\lambda} \operatorname{sinc}\left(\Omega \frac{L}{c}\right)} \cdot h_0 \sin\left[\Omega\left(t-\frac{L}{c}\right)\right] \qquad (5)$$

$$\operatorname{sinc}(x) = \frac{\sin(x)}{x}$$

$$\boxed{\delta\varphi = 2\pi \cdot \frac{L}{\lambda} \cdot \operatorname{sinc}\left(\Omega \frac{L}{c}\right) \cdot h_0 \sin\left[\Omega\left(t-\frac{L}{c}\right)\right]} \qquad (6)$$

Since $\delta\varphi \ll 1$ then

$$E_{OUT}^{(x)} = \frac{E_0}{2} e^{-i\omega\left(t - \frac{2L_x}{c}\right)} \cdot \left[1 + i\, G \cdot h_0 \cdot \sin\left[\Omega\left(t - \frac{L}{c}\right)\right]\right]$$

$$= \frac{E_0}{2} e^{-i(\omega t - \alpha)} \cdot \left\{1 + i G h_0 \frac{e^{i(\Omega t - \gamma)} - e^{-i(\Omega t - \gamma)}}{2i}\right\} =$$

$$= \frac{E_0}{2} \left\{ e^{-i(\omega t - \alpha)} + \right.$$

$$1 \quad + G \frac{h_0}{2} e^{-i[(\omega - \Omega)t - \alpha + \gamma]} +$$

$$2 \quad \left. - G \frac{h_0}{2} e^{-i[(\omega + \Omega)t - \alpha - \gamma]} \right\} \quad (7)$$

where G is the "gain" of the interferometer:

$$G = 2\pi \frac{L}{\lambda} \cdot \mathrm{sinc}\left(\Omega \frac{L}{c}\right) \quad (8)$$

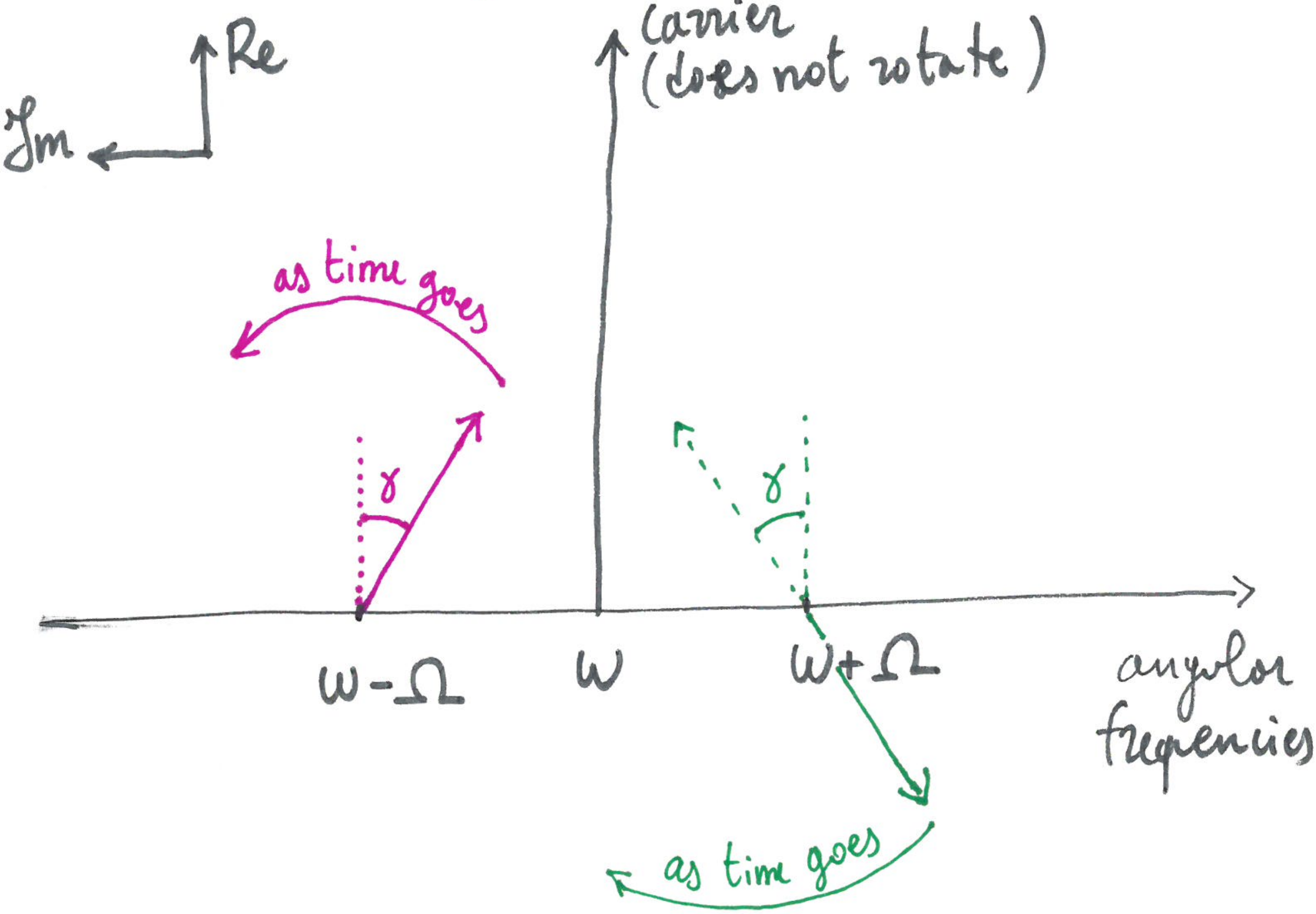
and γ a generic phase of the GW. For example γ allows changing the GW from a sine to a cosine:

$\gamma = \Omega \frac{L}{c} \rightarrow$ sine $\qquad \gamma = \Omega \frac{L}{c} + \frac{\pi}{2} \rightarrow$ cosine

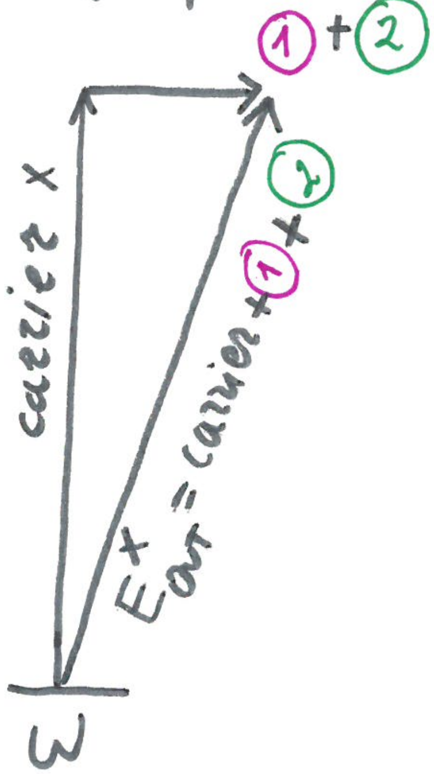
$e^{-i(\omega t-\alpha)}$ is the "carrier" and we take it as reference, hence it will have a constant real amplitude.

1 is the lower side band. It will turn w.r.t. the carrier following the function $\exp[i\Omega t]$

2 is the upper side band. It will turn w.r.t. the carrier following the function $\exp[-i\Omega t]$



As time goes the sum of the 2 side bands will never have a real component. They always produce an imaginary contribution:



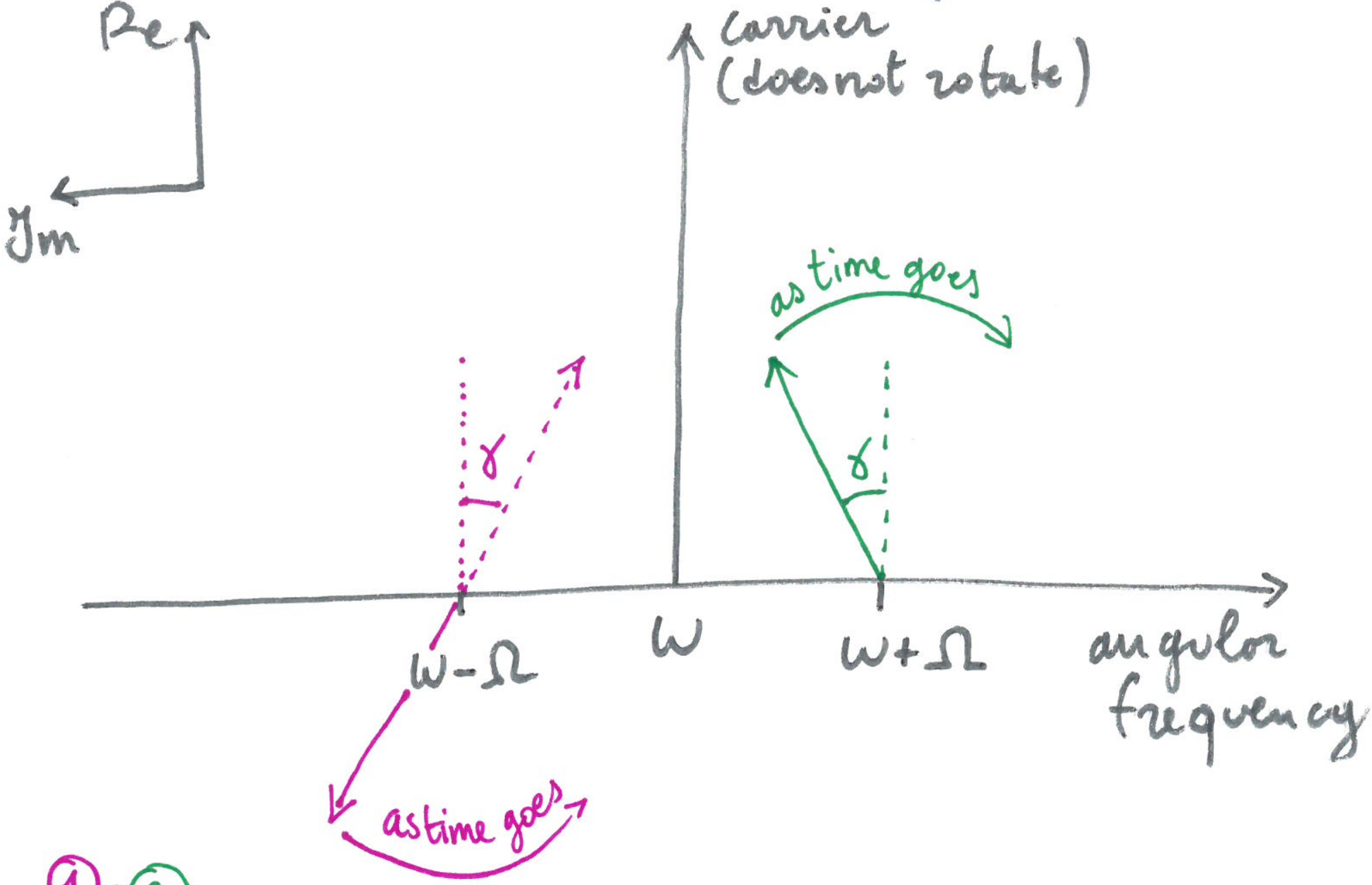
Let's see what happens to E^{Y}_{OUT}. The carrier has a phase $\beta = \omega \frac{2L_Y}{c}$ but the phase related to the GW has to be the same γ:

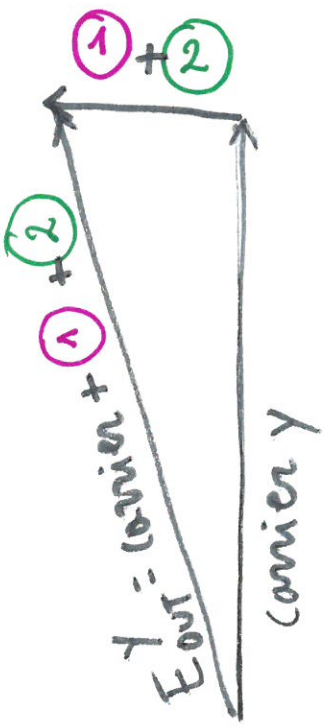
$$\delta\varphi = G\, h_0 \sin\left[\Omega t - \gamma\right]$$

Hence, the equivalent of eq.(7) is:

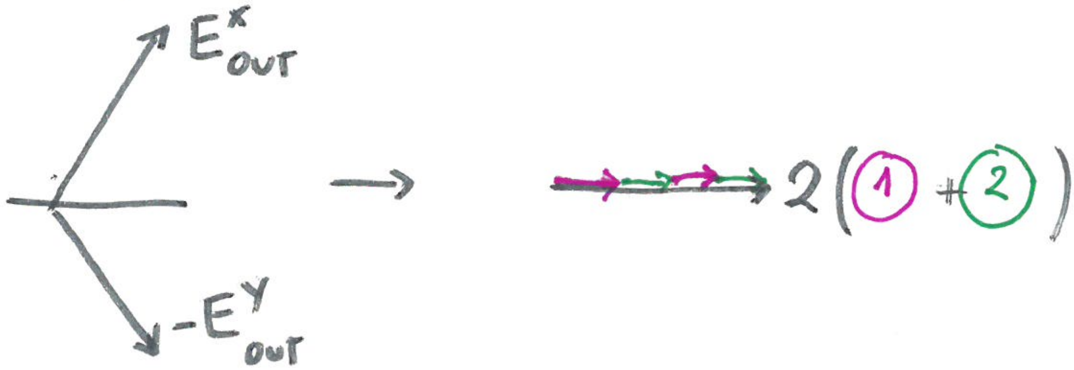
$$E^{(y)}_{OUT} = \frac{E_0}{2}\Big\{ e^{-i(\omega t - \beta)} + \text{①}\ - G\frac{h_0}{2} e^{-i\left[(\omega-\Omega)t - \beta + \gamma\right]} + \text{②}\ + G\frac{h_0}{2} e^{-i\left[(\omega+\Omega)t - \beta - \gamma\right]}\Big\}$$

As before we can do the phasor plot.





Considering the total output we have $E_{OUT} = E^{X}_{OUT} - E^{Y}_{OUT}$ hence, if the arm lengths are $\sim$ the same $\alpha \sim \beta$ and:



THE GW SIGNAL IS IN QUADRATURE WITH RESPECT TO THE CARRIER